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## EFFICIENCY INDICES OF INVESTMENT IN IT PROJECTS WITH UNEQUAL LIVES

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**Abstract.** Theoretical results not always give an unambiguous answer regarding the preference of using the indices of IT project investment efficiency. To complement some of such results, the Net Present Value (NPV), Profitability (PI), Equivalent Annual NPV (EANPV), Equivalent Annual PI (EAPI) and Internal Rate of Return (IRR) indices are researched by computer simulation. In this aim, the respective general problem is formulated, a model of comparative analysis of projects with unequal lives is defined and the SIMINV application is made up. Using SIMINV, the percentage of cases when the solutions, obtained using indices of each of the pairs  $\{NPV, PI\} - NP$ ,  $\{NPV, IRR\} - NR$ ,  $\{PI, IRR\} - PR$ ,  $\{EANPV, EAPI\} - NPE$ ,  $\{EANPV, IRR\} - NRE$  and  $\{EAPI, IRR\} - PRE$ , differ, for seven groups of alternatives of initial data, is determined. Based on done calculations were identified some properties of indices, including: the character of dependences on initial data; the relation larger/smaller between percentages of each of the pairs  $\{NP, NPE\}$ ,  $\{NR, NRE\}$  and  $\{PR, PRE\}$  (for example, the use of EANPV and EAPI indices to compare projects with unequal lives not only allows a more accurate estimation of projects efficiency, but also the solutions obtained may differ more frequently than when using the NPV and PI indices); the overall size of the value range and maximum average percentage of cases with different solutions, which is of approx. 57% for the pair of indices EANPV and EAPI and is between 18% and 52% for the other five index pairs specified above.

**Keywords:** investment projects, comparative analysis, net present value, profitability index, internal rate of return, equivalent annual value method, computer simulation.

### 1. Introduction

As is well known, the efficient computerization essentially contributes to the economic development and the society prosperity. Offered advantages impose the computerization of diverse activities implying respective investments. A decision of investment in an IT project is usually made on the basis of efficiency criteria/indices.

In economic analysis of IT projects (i-projects), the choice of indices to estimate the solution alternatives is of prime importance. For the assessment of economic efficiency of investment projects, in various sources is recommended to use such indicators as: profit, profit rate [1, 2], payback period on investment, net present value [1, 3, 4], profitability index [1, 4, 5], internal rate of return [1-3, 5], return on investment [1, 6], economic return on investments [2, 6], adjusted expenditure [6], total costs of ownership [7] and so on.

Depending on project product and its field of use, the set of applied indices may differ. In a specific project, a small set of indices is usually applied. It is



recommended to analyze  $7 \pm 2$  indices in [6]. Typically, 1-3 core indices and a few auxiliary indices are used. Moreover, the investment is usually decided based on an optimization problem. It contains one optimization criterion, and the other factors are taken into account as constraints. Thus, from the multitude of indices, a successful index as optimization criterion is important to choose (see, for example, [8]).

The known theoretical results do not give an unambiguous answer on preferences of using, in concrete situations, the indices for the estimation of efficiency of investment in i-projects. In order to extend the theoretical results in the field, in the paper some such indices are researched comparatively by computer simulation.

## 2. Preliminary considerations

Adequate guidance on the multitude of investment project estimation indices can lead to more successful solutions in the field. Related research is conducted in [2-10] and others. The most synthetic index of the efficiency of economic activity of an economic agent as a whole is considered the rate of profit [2]. In methodological recommendations [3], the basic indices for estimating the economic efficiency of investment projects are: net value, net present value, internal rate of return, rate of return on investments, payback period on investments and indices that characterize the financial state of the enterprise participating in the project. The most commonly used indices for valuing an investment are, according to [10], the net present value, the internal rate of return, the profitability index (the ratio of benefits to costs) and the payback period. The World Bank's financial criteria for choosing an investment project are: payback period on investment, net present value, discount rate, internal rate of return, rate of profitability and the profitability index. They are also of interest such indices commonly used as [2, 3, 6]: economic return on investment, adjusted expenditure, capital commitment (total discounted costs), global cost, net profit, revenue/cost ratio and the profitability index.

The multitude of efficiency criteria for investment in i-projects is caused by the diversity of aspects that characterize the respective situation-problems. As mentioned in Section 1, the optimization problem of investment in a specific project contains one optimization criterion, and the other factors are considered as constraints. The optimization criterion itself may be a composite one, comprising several indices with a certain weight. As constraints, when creating i-products, they often use: the maximum admissible amount of investments, the minimum allowed payback period on investment, and so on.

From the multitude of indices, in [11] are selected and described 16, most commonly used for estimating the economic efficiency of i-products, namely: profit, profit rate, discounted return on investment ( $R_d$ ), payback period on investments, updated payback period on investments, economic return on investments (general index of economic efficiency of investments) -  $R^{EI}$ , net value, net present value (NPV), internal rate of return (IRR), accounting rate of return, profitability index (PI), annual economic effect, annual adjusted expenditure, adjusted expenditure ( $C^{EN}$ ), total cost of ownership (TCO) and annual average costs of ownership.

The comparative analysis, performed in [12] and based on correlation between indices, the specificity of the time value of money, the different duration of projects



and also the range and importance of the characterized aspects, led to the reduction of the number of core indices for the comparative analysis of *i*-projects from 16 to 7, namely:  $R_d$ ,  $R^{EI}$ , NPV, IRR, PI,  $C^{EN}$  and TCO, eventually in conjunction with the equivalent annual value (EAV) method.

Moreover, according to Statement 2 of [8], for projects, the revenues from the implementation of which can be estimated with reasonable efforts, the use of  $C^{EN}$ , TCO,  $R_d$  and  $R^{EI}$  indices as basic indices of economic efficiency is not appropriate. Thus, out of the 16 mentioned above, as basic indices, for projects the revenues from the implementation of which can be estimated with reasonable efforts, remained three: NPV, IRR and PI, eventually in conjunction with the EAV method.

### 3. The general problem of comparing the efficiency indices

As is well known, the EAV method [13] is used for the appropriate comparison of projects with different lifetimes. It puts in an adequate correspondence to the updated summary value over a period of time of an index of a value over a shorter period, e.g. one year, thus allowing comparative analysis of projects with different lifetimes of their products. It is based on the capital recovery factor (CRF), which represents the ratio between a constant annuity and the discounted value of the receiver of this annuity for a certain period of time. The CRF can be interpreted as the value to be received each year during the product use, so that the actual total value of all these equal payments is equivalent to an one current monetary unit payment.

In case of discount rate  $d$  and duration of product use  $D$ , the CRF value is determined as [13]

$$CRF = \left[ \sum_{t=1}^D \frac{1}{(1+d)^t} \right]^{-1} = \frac{d(1+d)^D}{(1+d)^D - 1}, \quad (1)$$

From this formula, one has  $CRF(D=1) = d + 1$  and  $CRF(D \rightarrow \infty) = d$ ; thus,  $d \leq CRF < d + 1$  [12]. For the index  $XX$ , which characterizes a certain absolute value for the entire period  $D$ , the equivalent annual value will be noted  $EAXX$  and is determined as

$$EAXX = CRF \times XX. \quad (2)$$

If the EAV method applies to the NPV index, it is also called the equivalent annual cost method (EAC) [13]. For example, between EAC and CRF indices, occurs the relation  $EAC = EANPV = CRF \times NPV$ . Let  $I$  are investments and  $CF_t$  are cash flows in year  $t$  related to the project. Then NPV, IRR and PI indices are determined as:

$$NPV = \sum_1^D \frac{CF_t}{(1+d)^t} - I^C, \quad \sum_1^D \frac{CF_t}{(1+IRR)^t} - I^C = 0, \quad PI = 1 + \frac{NPV}{I^C}. \quad (3)-(5)$$

These three indices (NPV, IRR and PI) form a Parreto set: no one of the three can always replace the use of one or two of the other indices, in sense of obtaining the same solutions when comparing investment projects. At the same time, there are particular cases when the use of two of the three indices for comparing two investment projects, leads to the same solution. It is of interest how frequently such cases take place. Let's compare two projects, 1 and 2, the revenues from the implementation of which can be estimated with reasonable efforts. Then, at  $NPV_1 > 0$



(projects with  $NPV < 0$  are not eligible) and the pairwise comparison of the three indices for projects 1 and 2, it was found that [8]:

- 1) the use of NPV and PI indices leads to the same solution, being preferable the project 1, if and only if  $PI_1 = PI_2 + \alpha$ ,  $\alpha > 0$  and  $\alpha > (I_2^C / I_1^C - 1)(PI_2 - 1)$ ;
- 2) if  $I_1^C < I_2^C$ ,  $PI_1 = PI_2 + \alpha$  and  $0 < \alpha < (I_2^C / I_1^C - 1)(PI_2 - 1)$ , then the use of PI and NPV indices leads to different solutions;
- 3) the use of PI and IRR indices leads to the same solution, being preferable the project 1, in the following two cases: (a)  $IRR_1 > d \geq IRR_2$ ; (b)  $D_1 \geq D_2$ ,  $IRR_1 > IRR_2$ ,  $CF_{1t} = CF_1$ ,  $t = \overline{1, D_1}$  and  $CF_{2t} = CF_2$ ,  $t = \overline{1, D_2}$ ;
- 4) the use of EAPI and IRR indices leads to the same solution, being preferable the project 1, in the following two cases: (a)  $IRR_1 > d \geq IRR_2$ ; (b)  $D_2 \geq D_1$ ,  $IRR_1 > IRR_2$ ,  $CF_{1t} = CF_1$ ,  $t = \overline{1, D_1}$  and  $CF_{2t} = CF_2$ ,  $t = \overline{1, D_2}$ ;
- 5) the use of EAPI and EANPV indices leads to the same solution, being preferable the project 1, if  $EAPI_1 = EAPI_2 + \beta$ ,  $\beta > 0$  and  $\beta > I_2^C (EAPI_2 - CRF_2) / I_1^C - (EAPI_2 - CRF_1)$ ;
- 6) the use of EAPI and EANPV indices leads to different solutions if  $I_1^C > I_2^C$ ,  $EAPI_1 = EAPI_2 + \beta$  and  $\beta < I_2^C (EAPI_2 - CRF_2) / I_1^C - (EAPI_2 - CRF_1)$ .

But these results, except cases (1), (2), (5) and (6), do not fully characterize the opportunity of using one or another index when comparing investment i-projects. For example, according to [8], it can be considered that:

- 7) at  $IRR_1 > IRR_2 > d$  and  $D_1 > D_2$ , the use of PI and IRR indices leads, *usually*, to the same solution;
- 8) at  $IRR_1 < IRR_2$  and  $D_1 > D_2$ , the use of EAPI and IRR indices leads, *usually*, to the same solution.

So, in conditions of items (7) and (8), there may be cases when the use of PI and IRR indices and, respectively, of EAPI and IRR indices leads to different solutions. It is of interest to know how often such cases take place. To this and other aspects, the answer can be obtained by computer simulation.

#### 4. A model for comparative analysis of projects with unequal lives

The general research problem is the following. They are compared two investment projects, 1 and 2, with different lifetimes  $D_1 > D_2$ . When updating the values of indices, as time reference point will be the projects launch in operation; this time is the same for projects 1 and 2. It is required to identify, by computer simulation, the percentage of cases when the solutions, obtained using indices of each of the pairs {NPV, PI} (NI) –  $q_{NP}$ , {NPV, IRR} (NR) –  $q_{NR}$ , {PI, IRR} (PR) –  $q_{PR}$ , {EANPV, EAPI} (ENP) –  $q_{ENI}$ , {EANPV, IRR} (ENR) –  $q_{ENR}$  and {EAPI, IRR} (EPR) –  $q_{EPR}$ , leads to different solutions.

The NPV, PI, IRR, EANPV and EAPI values are determined according to formulas (1)-(5). The discount rate  $d$  will be considered constant and equal for the two projects, but the values of  $CF_t$  and also those of  $I$  and  $D$  can be different for the two projects. They are also introduced two parameters,  $g$  and  $v$ . Parameter  $g$  value is



determined for reasons of ensuring a given value  $r$  for the IRR index. So, from (4) at  $CF_t = CF$ ,  $t = 1, 2, \dots, D$ , one has

$$\sum_{t=1}^D \frac{CF_t}{(1+r)^t} - I = CF \sum_{t=1}^D \frac{1}{(1+r)^t} - I = CF \frac{1-(1+r)^{-D}}{r} - I = CF/g - I = 0$$

that is  $g = CF/I = r/[1 - (1+r)^{-D}]$ . Thus,  $g$  depends on  $r$  and  $D$  and, at the same time, it establishes the relation between the value  $I$  of investment and the average value  $CF$  of cash flows  $CF_t$ ,  $t = 1, 2, \dots, D$ . Of course, at  $CF_t \neq CF$ ,  $t = 1, 2, \dots, D$  the IRR value isn't equal to  $r$ , but it is relatively close to it.

In its turn, parameter  $\nu$  characterizes the range of relative variation of  $CF_t$  with respect to  $CF$ . Therefore, the value of  $\nu$  is assigned according to the value  $CF = gI$ , namely  $\nu = (CF - CF_{\min})/CF = (CF_{\max} - CF)/CF$ . So,  $CF_{\min} = CF(1 - \nu) = gI(1 - \nu)$ ,  $CF_{\max} = CF(1 + \nu) = gI(1 + \nu)$  and  $CF_t \in [CF_{\min}; CF_{\max}]$ ,  $t = 1, 2, \dots, D$ .

For concrete calculations, it is needed to know the reasonable values for  $d$ ,  $r$  and  $\nu$ . For  $\nu$  it will be used the values of the range  $[0.1; 0.9]$ , that is  $\nu \in [0.1; 0.9]$ .

Usually, **the discount rate  $d$**  value is established equal to the weighted average cost of capital (WACC), or the historical average returns of a similar project. According to:

- a) estimate [14], the Apple's WACC is 11.7%;
- b) KPMG Cost of Capital Study 2021 [15], in the period of 2005-2021 years WACC value was in the range of 6.6-8.9%. The highest WACC was observed in the Technology (8.9%), Automotive (7.6%) and Industrial manufacturing (7.5%);
- c) David Turney [16], the overall publicly traded equities market discount rate was estimated to be approximately 5.81% (January 2018); also, the estimated WACC range for the privately-held building materials company was 10% to 12%;
- d) [17] the Total market (7229 firms) WACC is 5.14%, Total market without financials (5169 firms) WACC is 5.75%, Software systems and applications market (375 firms) WACC is 6.15% and Semiconductor equipment market (34 firms) WACC is 6.95%.

Taking into account these data, in calculations is reasonable to use the **discount rate  $d \in [0.05; 0.14]$** .

With refer to the value of **internal rate of return IRR =  $r$** , according to:

- a) Industry Ventures [18], start-up companies should target an IRR of at least 30%, later stage companies - an IRR of 20%, and growth venture funds - an IRR of 12-18%;
- b) Angel Resource Institute [19] study, based on 136 complete investments in the period of 2010-2016 years, the overall IRR is approximately 22%;
- c) Properety Club [20], an IRR of **20%** would be considered good, but it's important to remember that it's always related to the cost of capital.

So, if  $IRR > WACC = d$ , then it is a good project, that is the lower limit for IRR values is determined by the lower limit for  $d$ . Taking into account these data, in calculations the range  $r \in [0.1; 1]$  for the IRR is reasonable to use.

In calculations, the duration  $D$  of investment projects will take values in the range of  $[1; 10]$  stages (years, etc.), that is  $D \in [1; 10]$ , and the investment  $I$  – in the



range of [100; 1000] conventional units, that is  $I \in [100; 1000]$ .

So, taking into account the ranges for  $d$ ,  $r$ ,  $v$ ,  $D$  and  $I$ , namely  $d \in [0.05; 0.14]$ ,  $r \in [0.1; 1]$ ,  $v \in [0.1; 0.5]$ ,  $D \in [1; 10]$  and  $I \in [100; 1000]$ , a very large number of alternatives of initial data can be used in calculations. From these, seven groups of alternatives, (1)-(7), are selected. In all of them, the  $CF_t$  values are generated randomly at uniform repartition in the respective range as follows:

$$CF_{1t} \in [CF_{1\min}; CF_{1\max}], \text{ where } CF_{1\min} = g(1 - v)I_1 \text{ and } CF_{1\max} = g(1 + v)I_1;$$

$$CF_{2t} \in [CF_{2\min}; CF_{2\max}], \text{ where } CF_{2\min} = g(1 - v)I_2 \text{ and } CF_{2\max} = g(1 + v)I_2.$$

In alternative (6), the values of  $I$  and  $D$  are also generated randomly at uniform repartition in the respective range:  $I_1 \in [100; 1000]$ ,  $I_2 \in [100; 1000]$ ,  $D_2 \in [1; 9]$  and  $D_1 \in [D_2+1; 10]$ . Moreover, in alternative (7) additionally the values of  $I$ ,  $D$ ,  $r$  and  $v$  are generated randomly in the respective range:  $I_1 \in [100; 1000]$ ,  $I_2 \in [100; 1000]$ ;  $D_2 \in [1; 9]$ ,  $D_1 \in [D_2+1; 10]$ ,  $r \in [0.1; 1.0]$  and  $v \in [0.1; 0.9]$ .

The seven groups of alternatives are (only cases for which  $NPV_1 > 0$ ,  $NPV_2 > 0$  are considered):

- 1) the reference group (dependence on  $d$ ):  $d = \{0.05, 0.06, 0.07, \dots, 0.14\}$ ;  $D_1 = 10$ ,  $D_2 = 5$ ;  $I_1 = 1000$ ,  $I_2 = 500$ ;  $r = 0.2$ ;  $v = 0.5$ ;
- 2) dependence on  $D_2$ :  $d = \{0.05, 0.06, 0.07, \dots, 0.14\}$ ;  $D_1 = 10$ ,  $D_2 = \{1, 2, 3, \dots, 9\}$ ;  $I_1 = 1000$ ,  $I_2 = 500$ ;  $r = 0.2$ ;  $v = 0.5$ ;
- 3) dependence on  $I_2$ :  $d = \{0.05, 0.06, 0.07, \dots, 0.14\}$ ;  $D_1 = 10$ ,  $D_2 = 5$ ;  $I_1 = 1000$ ,  $I_2 = \{100, 200, 300, \dots, 900, 1000\}$ ;  $r = 0.2$ ;  $v = 0.5$ ;
- 4) dependence on  $r$ :  $d = \{0.05, 0.06, 0.07, \dots, 0.14\}$ ;  $D_1 = 10$ ,  $D_2 = 5$ ;  $I_1 = 1000$ ,  $I_2 = 500$ ;  $r = \{0.1, 0.2, 0.3, \dots, 1.0\}$ ;  $v = 0.5$ ;
- 5) dependence on  $v$ :  $d = \{0.05, 0.06, 0.07, \dots, 0.14\}$ ;  $D_1 = 10$ ,  $D_2 = 5$ ;  $I_1 = 1000$ ,  $I_2 = 500$ ;  $r = 0.2$ ;  $v = \{0.1, 0.2, 0.3, \dots, 0.9\}$ ;
- 6) dependence on  $d^+$  (on  $d$  when  $D_2$  and  $I_2$  are generated randomly – partial general group):  $d = \{0.05, 0.06, 0.07, \dots, 0.14\}$ ;  $D_2 \in [1; 9]$ ,  $D_1 \in [D_2+1; 10]$ ;  $I_1 \in [100; 1000]$ ,  $I_2 \in [100; 1000]$ ;  $r = 0.2$ ;  $v = 0.5$ ;
- 7) dependence on  $d$  (the general group):  $d = \{0.05, 0.06, 0.07, \dots, 0.14\}$ ;  $D_2 \in [1; 9]$ ,  $D_1 \in [D_2+1; 10]$ ;  $I_1 \in [100; 1000]$ ,  $I_2 \in [100; 1000]$ ;  $r \in [0.1; 1.0]$ ;  $v \in [0.1; 0.9]$ .

For each of the seven alternatives, the respective percentages  $q_{NP}$ ,  $q_{NR}$ ,  $q_{PR}$ ,  $q_{NPE}$ ,  $q_{NRE}$ ,  $q_{PRE}$  and  $f$  have to be determined. Here  $f$  is the dependence on respective parameter (parameters) of the percentage of generated cases of initial data for which  $NPV_1 < 0$  or  $NPV_2 < 0$  or  $NPV_1 < 0$  and  $NPV_2 < 0$  (failure cases). The respective computer simulation algorithm is described in Section 5.

### 5. Computer simulation algorithm

The algorithm, for the determination of percentages  $q_{NP}(d)$ ,  $q_{NR}(d)$ ,  $q_{PR}(d)$ ,  $q_{ENP}(d)$ ,  $q_{ENR}(d)$ ,  $q_{EPR}(d)$  and  $f(d)$  in general case (7), is the following.

1. Initial data:  $d_0$ ,  $\Delta d = 0.01$ ;  $r$ ,  $v$ ;  $D_{\min}$ ,  $D_{\max}$ ;  $I_{\min}$ ,  $I_{\max}$ ;  $N$  (total number of values for  $d$ ),  $K$  (total number of initial data values for the done value of  $d$  - sample size).
2.  $n := 1$ ,  $d := d_0$ .
3.  $m_f := 0$ ,  $m_{NP} := 0$ ,  $m_{NR} := 0$ ,  $m_{PR} := 0$ ,  $m_{NPE} := 0$ ,  $m_{NRE} := 0$ , and  $m_{PRE} := 0$ .
4.  $k := 1$ .



5. Generation, at uniform random distribution, of the values of quantities  $D_2 \in [D_{\min}; D_{\max} - 1]$ ,  $D_1 \in [D_2 + 1; D_{\max}]$ ,  $I_1 \in [I_{\min}; I_{\max}]$ , and  $I_2 \in [I_{\min}; I_{\max}]$  and determination of  $g_1 = r/[1 - (1 + r)^{-D_1}]$  and  $g_2 = r/[1 - (1 + r)^{-D_2}]$ .
6.  $CF_{1\min} := g_1(1 - v)I_1$ ,  $CF_{1\max} := g_1(1 + v)I_1$ ,  $CF_{2\min} := g_2(1 - v)I_2$ ,  $CF_{2\max} := g_2(1 + v)I_2$  and generation, at uniform random distribution, of the values of quantities  $CF_{1t} \in [CF_{1\min}; CF_{1\max}]$ ,  $t = 1, 2, \dots, D_1$  and  $CF_{2t} \in [CF_{2\min}; CF_{2\max}]$ ,  $t = 1, 2, \dots, D_2$ .
7. Determination of  $NPV_1$  according to (3). If  $NPV_1 < 0$ , then  $m_f := m_f + 1$  and go to Step 11.
8. Determination of  $NPV_2$  according to (3). If  $NPV_2 < 0$ , then  $m_f := m_f + 1$  and go to Step 11.
9. Determination of  $PI_1$ ,  $IRR_1$ ,  $EANPV_1$ ,  $EAPI_1$ ,  $NPV_2$ ,  $PI_2$ ,  $IRR_2$ ,  $EANPV_2$ , and  $EAPI_2$  taking into account the formulas (1)-(5).
10. Identification and counting the numbers  $m_{NP}$ ,  $m_{NR}$ ,  $m_{PR}$ ,  $m_{NPE}$ ,  $m_{NRE}$  and  $m_{PRE}$  of cases when the solutions, obtained using indices of each of the pairs NP, NR, PR, NPE, NRE and PRE, leads to different solutions.
11. If  $k < K$ , then  $k := k + 1$  and go to Step 5.
12.  $q_{NP}(d) := 100m_{NP}/(K - m_f)$ ,  $q_{NR}(d) := 100m_{NR}/(K - m_f)$ ,  $q_{PR}(d) := 100m_{PR}/(K - m_f)$ ,  $q_{NPE}(d) := 100m_{NPE}/(K - m_f)$ ,  $q_{NRE}(d) := 100m_{NRE}/(K - m_f)$ ,  $q_{PRE}(d) := 100m_{PRE}/(K - m_f)$  and  $f(d) := 100m_f/(K - m_f)$ .
13. If  $n < N$ , then  $d := d + \Delta d$  and go to Step 3.
14. Taking over the simulation results. Stop.

Similar, with respective adaptations, are the algorithms for the groups of alternatives (1)-(6).

## 6. Some results of performed calculations

To extend theoretical results in the domain by computer simulation, the application SIMINV in C++ was made up and used. Some of the obtained results are described in this section. Each set of initial data characterizes two concrete projects, 1 and 2. According to the algorithm described in Section 5 and the seven groups of alternatives specified in Section 4, a sample of 100000 was generated. So, were generated, for the group of alternatives:

- (1), (6) and (7) by  $10 \times 10^5 = 1$  mil sets of initial data;
- (2) and (5) by  $10 \times 9 \times 10^5 = 9$  mil sets of initial data;
- (3) and (4) by  $10 \times 10 \times 10^5 = 10$  mil sets of initial data.

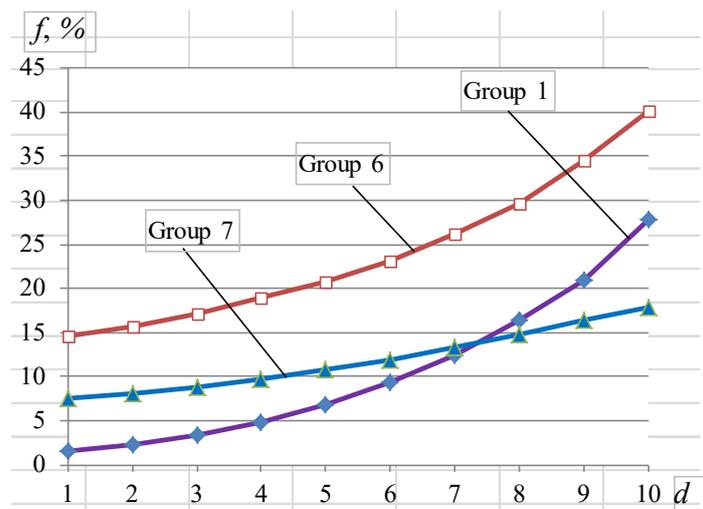
### 6.1. The number of initial data generation failures

The approach, used to establish and generate initial data sets, doesn't ensure the requirements of  $NPV_1 > 0$  and  $NPV_2 > 0$ . That is why the algorithm counters the total number of cases of failure ( $NPV_1 < 0$  or  $NPV_2 < 0$  or both  $NPV_1 < 0$  and  $NPV_2 < 0$ ). This number is used when calculating the values of percentages  $q_{NP}(\cdot)$ ,  $q_{NR}(\cdot)$ ,  $q_{PR}(\cdot)$ ,  $q_{NPE}(\cdot)$ ,  $q_{NRE}(\cdot)$  and  $q_{PRE}(\cdot)$ . But if this number is too large, then the calculation errors are also significant. Therefore it is important to know its value.

In Figure 1, the dependences of  $f$  on  $d$  for the groups of alternatives of initial data (1), (6) and (7) are shown. These dependencies are relatively close to each other and are increasing on  $d$ , but not overpassing 40.2 %. The results of performed calculations show also that for the group of alternatives of initial data:



- (2) the dependence  $f(d, D_2)$  is decreasing both on  $d$  and  $D_2$ , the range of values being  $[0.03; 44.1]$  % at  $d = 0.08$  and overall  $[0.00; 51.3]$ %;
- (3) the dependence  $f(d, I_2)$  is increasing on  $d$  and is very little dependent on  $I_2$ , the range of values being  $[4.77; 4.94]$ % at  $d = 0.08$  and overall  $[1.51; 27.5]$ %;
- (4) the dependence  $f(d, r)$  is decreasing both on  $d$  and  $r$ , but  $f(d, r) = 0$  at  $r \geq 0.5$ , the range of values being  $[0; 60.5]$  % at  $d = 0.08$  and overall  $[0; 99.65]$  %, but  $[0; 27.7]$  % at  $r \geq 0.2$ ;
- (5) the dependence  $f(d, v)$  is increasing both on  $d$  and  $v$ , but  $f(d, r) = 0$  at  $v = 0.1$ , the range of values being  $[0; 23.6]$  % at  $d = 0.08$  and overall  $[0; 51.8]$ %.



**Figure 1 – Percentages of cases of failure when generating the sets of initial data**

So, for all seven groups (1)-(7) of alternatives of initial data the dependences  $f(\cdot)$  are increasing or slightly increasing on  $d$ , the overall range of values being of  $[0; 51.8]$  %, except the case of group 4 at  $r = 0.1$  when the high limit is of 99.65%. Thus, in case of group 4 at  $r = 0.1$ , the sample of initial data is of  $100000(100 - 99.65)/100 = 350$  alternatives and can be insufficient. In all other cases, the sample of initial data overpasses  $100000(100 - 51.8)/100 = 48200$  alternatives and is sufficient.

**6.2. Examples which confirm the veracity of items (7) and (8) of Section 3**

To identify, if really, at conditions of items (7) and (8) of Section 3, there may be cases when the use of PI and IRR indices and, respectively, of EAPI and IRR indices leads to different solutions, were performed respective calculations using the SIMINV application. Some of such cases are described below in this section.

**Example 1** (using IRR and PI indices). Initial data common to both projects (1 and 2):  $d = 0.1$ . Project 1 is characterized by the following data:  $D_1 = 7, I_1 = 125.7, CF_{1,1} = 81.6, CF_{1,2} = 44.3, CF_{1,3} = 40.4, CF_{1,4} = 78.4, CF_{1,5} = 63.3, CF_{1,6} = 39.7, CF_{1,7} = 42.7$ . Also, Project 2 is characterized by data:  $D_2 = 6, I_2 = 609.0, CF_{2,1} = 206.8, CF_{2,2} = 407.0, CF_{2,3} = 250.2, CF_{2,4} = 305.0, CF_{2,5} = 412.6, CF_{2,6} = 385.5$ .

The results of calculations for indices IRR and PI are:  $PI_1 = 2.214, PI_2 = 2.290, IRR_1 = 0.450$  and  $IRR_2 = 0.436$ . So:  $PI_1 = 2.214 < PI_2 = 2.290$  and  $IRR_1 = 0.450 > IRR_2 = 0.436$ . Thus, the solutions obtained differ: according to the IRR index, one has to prefer the project 1, but according to the PI index, one has to prefer the project



2. This confirms the “usually” stipulation in item (7) of Section 3.

**Example 2** (using IRR and EAPI indices). Initial data common to both projects (1 and 2):  $d = 0.1$ . Project 1 is characterized by the following data:  $D_1 = 9, I_1 = 158, CF_{1,1} = 64.7, CF_{1,2} = 73.7, CF_{1,3} = 81.1, CF_{1,4} = 85.8, CF_{1,5} = 79.3, CF_{1,6} = 73.2, CF_{1,7} = 85.7, CF_{1,8} = 84.1, CF_{1,7} = 81.1$ . Also, Project 2 is characterized by the following data:  $D_2 = 7, I_2 = 691.1, CF_{2,1} = 382.1, CF_{2,2} = 308.8, CF_{2,3} = 392.6, CF_{2,4} = 381.7, CF_{2,5} = 274.4, CF_{2,6} = 275.9, CF_{2,7} = 321.2$ .

The results of calculations for indices IRR and PI are:  $EAPI_1 = 0.492, EAPI_2 = 0.490, IRR_1 = 0.454$  and  $IRR_2 = 0.475$ . So:  $EAPI_1 = 0.492 > EAPI_2 = 0.490$  and  $IRR_1 = 0.454 < IRR_2 = 0.475$ . Thus the solutions obtained differ: according to the EAPI index, one has to prefer the project 1, but according to the IRR index, one has to prefer the project 2. This confirms the “usually” stipulation in item (8) of Section 3.

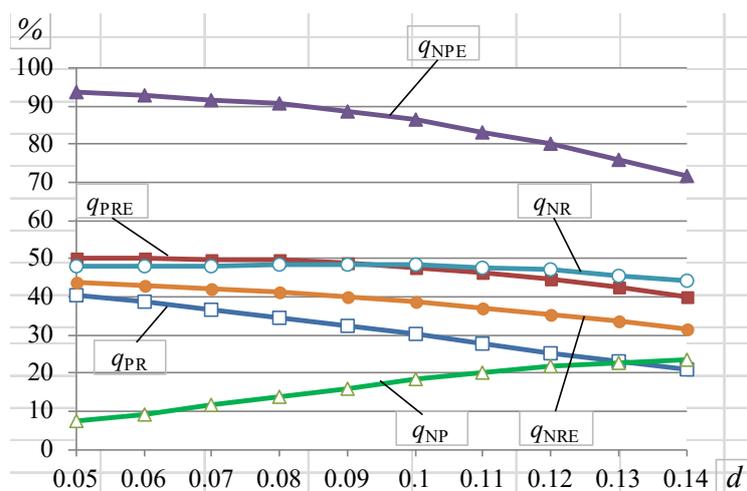
Thus, they are confirmed the fact that there may be cases when the use of PI and IRR indices and, respectively, of EAPI and IRR indices leads to different solutions. Let’s go further to determine the percentages  $q_{NP}, q_{NR}, q_{PR}, q_{NPE}, q_{NRE}$  and  $q_{PRE}$  of cases for which the obtained solutions differ.

### 6.3. Frequency of cases for which the obtained solutions differ

Computer simulation using the SIMINV application was performed for all seven groups of alternatives defined in Section 4. Some results are described below.

**The group of alternatives 1 - dependence on  $d$ .** Initial data:  $d = \{0.05, 0.06, 0.07, \dots, 0.14\}; D_1 = 10, D_2 = 5; I_1 = 1000, I_2 = 500; r = 0.2; v = 0.5$ . The obtained dependences  $q_{NP}(d), q_{NR}(d), q_{PR}(d), q_{NPE}(d), q_{NRE}(d)$  and  $q_{PRE}(d)$  are shown in Figure 2.

Figure 2 shows that all mentioned dependences, except the  $q_{NP}(d)$  one, are decreasing on  $d$ ; with refer to the  $q_{NP}(d)$  dependence, it is increasing on  $d$ . Also, by pairs, the largest discrepancy is between  $q_{NP}(d)$  and  $q_{NPE}(d)$ , follows the pair  $\{q_{PR}(d), q_{PRE}(d)\}$ , and the lowest discrepancy is between percentages  $q_{NR}(d)$  and  $q_{NRE}(d)$ . At the same time, one has  $q_{NP}(d) < q_{NPE}(d)$  and  $q_{PR}(d) < q_{PRE}(d)$ , but  $q_{NR}(d) > q_{NRE}(d)$ .



**Figure 2 – Percentages  $q_{NP}(d), q_{NR}(d), q_{PR}(d), q_{NPE}(d), q_{NRE}(d)$  and  $q_{PRE}(d)$**

The obtained ranges of values for the six dependences at  $d \in [0.05; 0.14]$  are specified in Table 1.

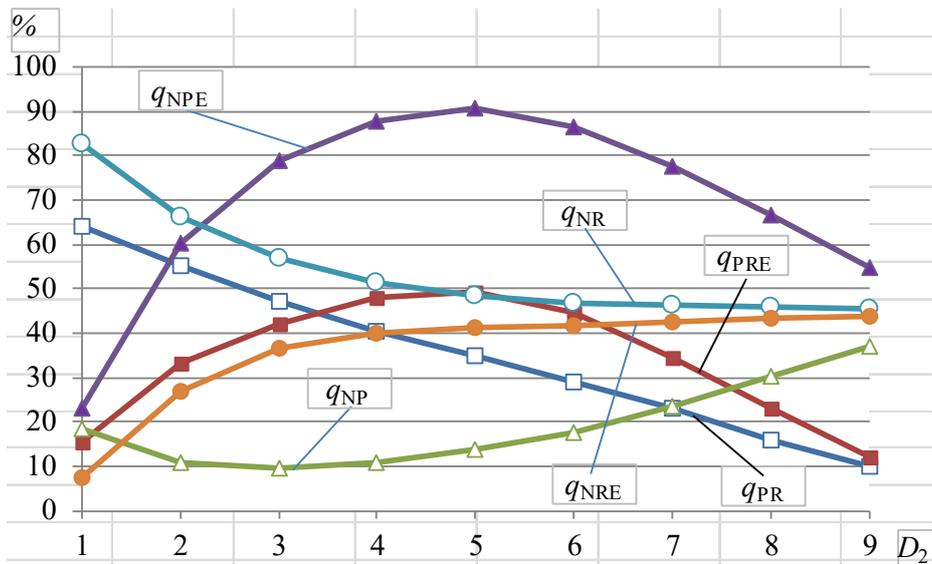


**Table 1 – The ranges of values for the six dependences on  $d \in [0.05; 0.14]$**

	$q_{NP}(d)$	$q_{NR}(d)$	$q_{PR}(d)$	$q_{NPE}(d)$	$q_{NRE}(d)$	$q_{PRE}(d)$
Minimum of $q(d)$	7.37	44.25	20.81	71.59	31.66	39.93
Maximum of $q(d)$	23.44	48.39	40.54	93.67	43.72	50.05

Based on data of Table 1, it can be concluded that, on average, exists a considerable number of cases ( $q_{NPE}(d) \in [71.59; 93.67]\%$ ) when the use of EANPV and EAPI indices leads to different solutions. The use of other pairs of compared indices also can lead to different solutions in a significant number of cases. The largest range (the difference between the high and low limits) is that of  $q_{NPE}(d)$  equal to  $93.67 - 71.49 = 22.18\%$  ( $q_{NPE}(d) \in [71.49; 93.67]\%$ ), and the narrowest range is that of  $q_{NR}(d)$  equal to  $48.39 - 44.25 = 4.14\%$  ( $q_{NP}(d) \in [44.25; 48.39]$ ).

**The group of alternatives 2 - dependence on  $D_2$ .** Initial data:  $d = \{0.05, 0.06, 0.07, \dots, 0.14\}$ ;  $D_1 = 10$ ,  $D_2 = \{1, 2, 3, \dots, 9\}$ ;  $I_1 = 1000$ ,  $I_2 = 500$ ;  $r = 0.2$ ;  $v = 0.5$ . In graphical form, the dependences  $q_{NP}(D_2)$ ,  $q_{NR}(D_2)$ ,  $q_{PR}(D_2)$ ,  $q_{NPE}(D_2)$ ,  $q_{NRE}(D_2)$  and  $q_{PRE}(D_2)$  at  $d = 0.08$  are presented in Figure 3.



**Figure 3 – Percentages  $q_{NP}(D_2)$ ,  $q_{NR}(D_2)$ ,  $q_{PR}(D_2)$ ,  $q_{NPE}(D_2)$ ,  $q_{NRE}(D_2)$  and  $q_{PRE}(D_2)$**

According to Figure 3, the character of the six dependences on  $D_2$  are different: that of  $q_{NRE}(D_2)$  is increasing; those of  $q_{NPE}(D_2)$  and  $q_{PRE}(D_2)$  initially are increasing and after are decreasing; that of  $q_{NP}(D_2)$  is decreasing at  $D_2 \leq 3$  and is increasing at  $D_2 > 3$ ; those of  $q_{NR}(D_2)$  and  $q_{PR}(D_2)$  are decreasing. Also, by pairs, the largest discrepancy is between  $q_{NP}(D_2)$  and  $q_{NPE}(D_2)$ . At the same time, one has  $q_{NP}(D_2) < q_{NPE}(D_2)$ ,  $q_{NR}(D_2) > q_{NRE}(D_2)$ , but  $q_{PR}(D_2) > q_{PRE}(D_2)$  at  $D_2 \leq 3$  and  $q_{PR}(D_2) < q_{PRE}(D_2)$  at  $D_2 > 3$ . The obtained ranges of values for the six dependences at  $d \in [0.05; 0.14]$  are specified in Table 2.

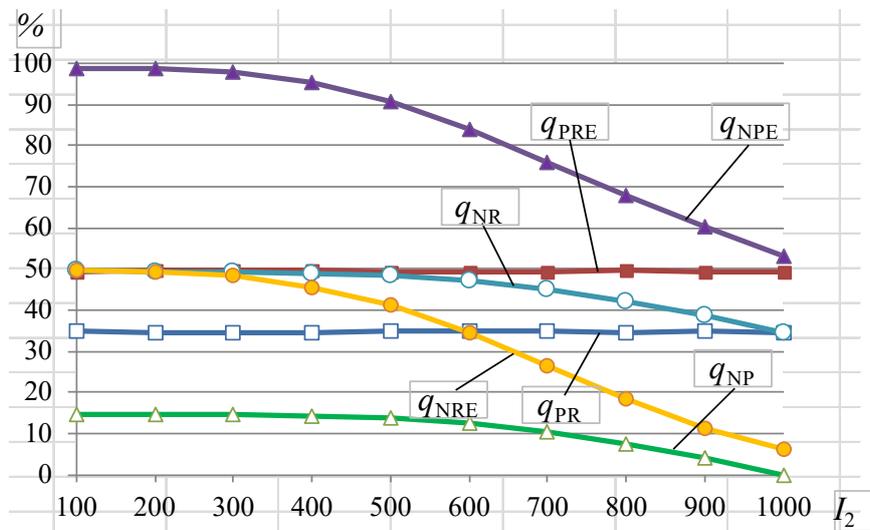
**Table 2 – The ranges of values for the six dependences on  $D_2$  at  $d \in [0.05; 0.14]$**

	$q_{NP}(D_2)$	$q_{NR}(D_2)$	$q_{PR}(D_2)$	$q_{NPE}(D_2)$	$q_{NRE}(D_2)$	$q_{PRE}(D_2)$
Minimum of $q(D_2)$	3.38	31.40	4.55	13.18	4.61	9.38
Maximum of $q(D_2)$	37.50	82.73	75.61	93.61	47.26	50.7458



It can be seen that there can be a large number of cases when the use of EANPV and EAPI indices leads to different solutions, which, on average, can reach 93.61%, this being approx. equal to the  $q(d)$ :  $\max\{q_{NPE}(d)\} = 93.67\% \approx \max\{q_{NPE}(D_2)\} = 93.61\%$ . The use of other pairs of compared indices also can lead to different solutions in a significant number of cases. The largest range is that of  $q_{NPE}(D_2)$  equal to  $93.61 - 13.18 = 80.43\%$  ( $q_{NPE}(D_2) \in [13.18; 93.61]\%$ ), and the narrowest range is that of  $q_{NP}(D_2)$  equal to  $37.50 - 3.38 = 34.12\%$  ( $q_{NR}(D_2) \in [3.38; 37.50]$ ).

**The group of alternatives 3 - dependence on  $I_2$ .** Initial data:  $d = \{0.05, 0.06, 0.07, \dots, 0.14\}$ ;  $D_1 = 10, D_2 = 5$ ;  $I_1 = 1000, I_2 = \{100, 200, 300, \dots, 900, 1000\}$ ;  $r = 0.2$ ;  $v = 0.5$ . Some results of calculations with refer to dependences  $q_{NP}(I_2), q_{NR}(I_2), q_{PR}(I_2), q_{NPE}(I_2), q_{NRE}(I_2)$  and  $q_{PRE}(I_2)$  at  $d = 0.08$  are shown in Figure 4.



**Figure 4 – Percentages  $q_{NP}(I_2), q_{NR}(I_2), q_{PR}(I_2), q_{NPE}(I_2), q_{NRE}(I_2)$  and  $q_{PRE}(I_2)$**

From Figure 4, one can see that percentages  $q_{PR}(I_2)$  and  $q_{PRE}(I_2)$  practically do not depend on  $I_2$ , while the other four dependences are decreasing on  $I_2$ . Also, the form of dependences  $q_{NPE}(I_2)$  and  $q_{NRE}(I_2)$  and those of  $q_{NP}(I_2)$  and  $q_{NR}(I_2)$  are very similar to each other. As in previous two groups of alternatives, the largest discrepancy is between  $q_{NP}(I_2)$  and  $q_{NPE}(I_2)$ . At the same time, at small values of  $I_2$ , the following equalities take place:  $q_{NR}(I_2) \approx q_{NRE}(I_2) \approx q_{PRE}(I_2)$ . To mention that  $q_{NP}(I_2) < q_{NPE}(I_2), q_{PR}(I_2) < q_{PRE}(I_2)$ , but  $q_{NR}(I_2) > q_{NRE}(I_2)$ . The obtained ranges of values for the six dependences at  $d \in [0.05; 0.14]$  are specified in Table 3.

**Table 3 – The ranges of values for the six dependences on  $I_2$  at  $d \in [0.05; 0.14]$**

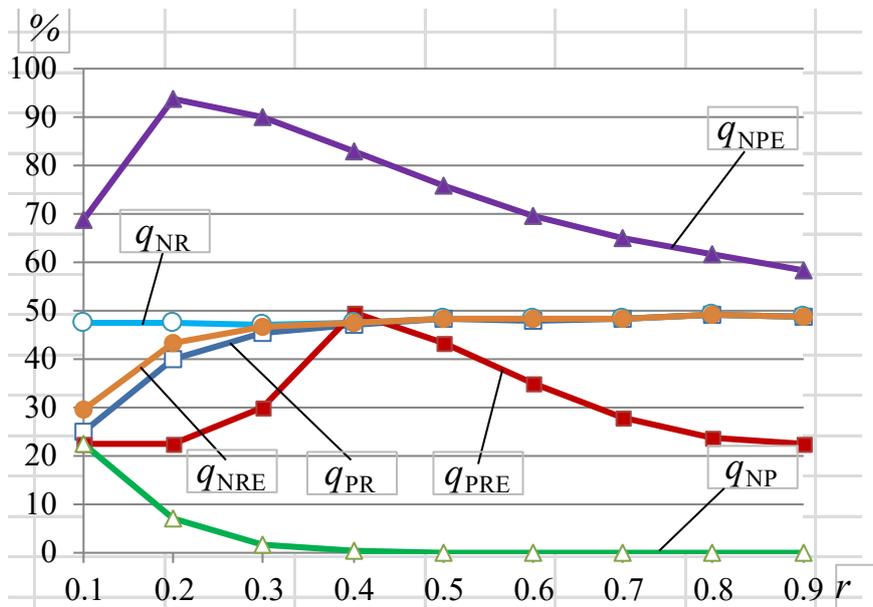
	$q_{NP}(I_2)$	$q_{NR}(I_2)$	$q_{PR}(I_2)$	$q_{NPE}(I_2)$	$q_{NRE}(I_2)$	$q_{PRE}(I_2)$
Minimum of $q(I_2)$	0	20.40	20.40	43.25	3.89	40.11
Maximum of $q(I_2)$	37.10	57.85	40.54	99.27	56.58	50.38

As in previous two groups of alternatives, there can be a considerable number of cases when the use of EANPV and EAPI indices leads to different solutions, which, on average, can reach 99.27%. The use of other pairs of compared indices also can lead to different solutions in a significant number of cases. The largest range is that of



$q_{NPE}(I_2)$  equal to  $99.27 - 43.25 = 56.02$ ), and the narrowest range is that of  $q_{PRE}(I_2)$  equal to  $50.38 - 40.11 = 10.27\%$  ( $q_{NR}(I_2) \in [40.11; 50.38]$ ). To mention that  $q_{NP}(I_2) = 0$  at  $I_2 = I_1$  no matter of the value of  $d \in [0.05; 0.14]$ .

**The group of alternatives 4 - dependence on  $r$ .** Initial data:  $d = \{0.05, 0.06, 0.07, \dots, 0.14\}$ ;  $D_1 = 10, D_2 = 5; I_1 = 1000, I_2 = 500; r = \{0.1, 0.2, 0.3, \dots, 1.0\}; v = 0.5$ . The obtained dependences  $q_{NP}(r), q_{NR}(r), q_{PR}(r), q_{NPE}(r), q_{NRE}(r)$  and  $q_{PRE}(r)$  at  $d = 0.08$  are shown in Figure 5.



**Figure 5 – Percentages  $q_{NP}(r), q_{NR}(r), q_{PR}(r), q_{NPE}(r), q_{NRE}(r)$  and  $q_{PRE}(r)$**

One can see that the character of dependencies  $q_{NPE}(r)$  and  $q_{PRE}(r)$  are similar: they are increasing at small values of  $r$  and are decreasing at large values of  $r$ . The dependencies  $q_{PR}(r)$  and  $q_{NRE}(r)$  practically coincide and are increasing on  $r$ . The dependence  $q_{NR}(r)$  very little depends on  $r$  approx. coinciding with the previous two at  $r \geq 0.4$ . The largest discrepancy is also between  $q_{NP}(r)$  and  $q_{NPE}(r)$ . At the same time, take place  $q_{NP}(r) < q_{NPE}(r)$  and  $q_{NR}(I_2) \geq q_{NRE}(I_2)$ . Also, usually  $q_{PR}(r) > q_{PRE}(r)$ .

The obtained ranges of values, for the six dependences at  $d \in [0.05; 0.14]$ , are systemized in Table 4.

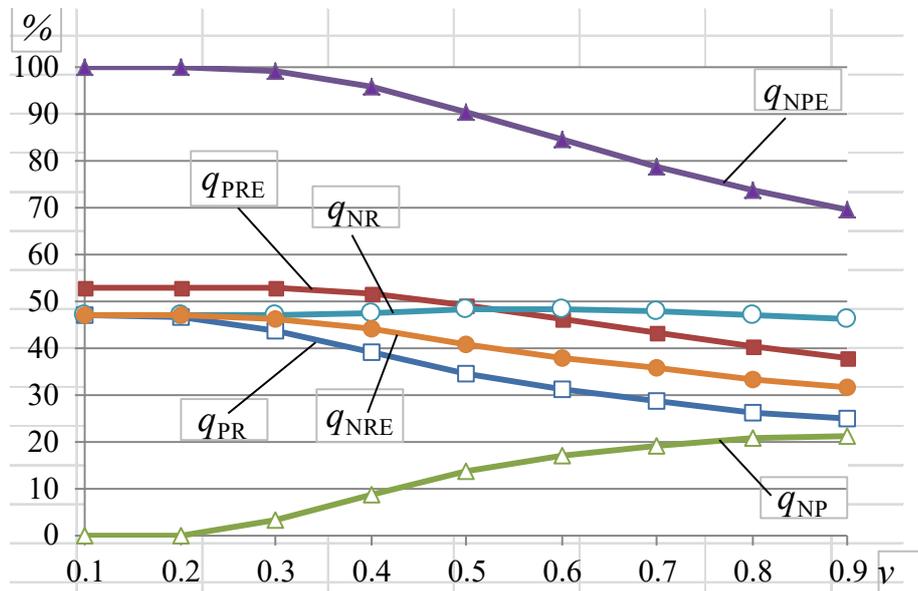
**Table 4 – The ranges of values for the six dependences on  $r$  at  $d \in [0.05; 0.14]$**

	$q_{NP}(r)$	$q_{NR}(r)$	$q_{PR}(r)$	$q_{NPE}(r)$	$q_{NRE}(r)$	$q_{PRE}(r)$
Minimum of $q(r)$	0.004	26.99	9.66	36.93	15.54	19.40
Maximum of $q(r)$	23.34	49.23	49.11	93.76	49.21	50.40

As in previous four groups of alternatives, there can be a considerable number of cases when the use of EANPV and EAPI indices leads to different solutions, which, on average, can reach 93.76 %. The use of other pairs of compared indices also can lead to different solutions in a significant number of cases. The largest range is that of  $q_{NPE}(r)$  equal to  $93.76 - 36.93 = 46.83\%$ , and the narrowest range is that of  $q_{NP}(r)$  equal to  $23.34 - 0.004 = 23.34\%$  ( $q_{NR}(I_2) \in [0.004; 23.34]$ ).



**The group of alternatives 5 - dependence on  $v$ .** Initial data:  $d = \{0.05, 0.06, 0.07, \dots, 0.14\}$ ;  $D_1 = 10, D_2 = 5$ ;  $I_1 = 1000, I_2 = 500$ ;  $r = 0.2$ ;  $v = \{0.1, 0.2, 0.3, \dots, 0.9\}$ . The obtained dependences  $q_{NP}(v), q_{NR}(v), q_{PR}(v), q_{NPE}(v), q_{NRE}(v)$  and  $q_{PRE}(v)$  at  $d = 0.08$  are shown in Figure 6.



**Figure 6 – Percentages  $q_{NP}(v), q_{NR}(v), q_{PR}(v), q_{NPE}(v), q_{NRE}(v)$  and  $q_{PRE}(v)$**

By their character, all six dependencies  $q(v)$  are very similar to the respective six dependencies  $q(d)$  (see Figure 2): four of the six dependencies, namely  $q_{PR}(v), q_{NPE}(v), q_{NRE}(v)$  and  $q_{PRE}(v)$  are decreasing on  $v$ ; with refer to the  $q_{NP}(v)$  dependence at  $d = 0.08$ , it is increasing at  $v \in [0.1; 0.8]$  and  $q_{NR}(v)$  very little depends on  $v$ . At the same time, at  $v \in [0.1; 0.2]$  take place  $q_{NR}(v) = q_{PR}(v) = q_{NRE}(v)$ . By pairs, the largest discrepancy is between  $q_{NP}(v)$  and  $q_{NPE}(v)$ , follows the pair  $\{q_{PR}(v), q_{PRE}(v)\}$ , and the lowest discrepancy is between percentages  $q_{NR}(v)$  and  $q_{NRE}(v)$ . Also, one has  $q_{NP}(v) < q_{NPE}(v)$  and  $q_{PR}(v) \leq q_{PRE}(v)$ , but  $q_{NR}(v) \geq q_{NRE}(v)$ . The obtained ranges of values for the six dependences are specified in Table 5.

**Table 5 – The ranges of values for the six dependences on  $v$  at  $d \in [0.05; 0.14]$**

	$q_{NP}(v)$	$q_{NR}(v)$	$q_{PR}(v)$	$q_{NPE}(v)$	$q_{NRE}(v)$	$q_{PRE}(v)$
Minimum of $q(v)$	0	38.89	16.31	59.85	25.86	33.99
Maximum of $q(v)$	22.97	48.94	47.43	100	47.43	53.26

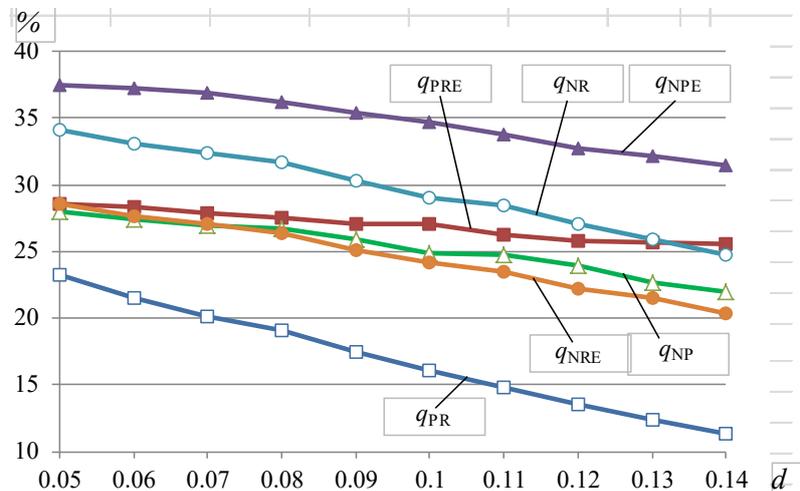
Based on data of Table 5, it can be concluded that, on average, there are a considerable number of cases ( $q_{NPE}(v) \in [59.85; 100]\%$ ) when the use of EANPV and EAPI indices leads to different solutions. The use of other pairs of compared indices also can lead to different solutions in a significant number of cases. The largest range is that of  $q_{NPE}(v)$  equal to  $100 - 59.85 = 40.15\%$ , and the narrowest range is that of  $q_{NR}(v)$  equal to  $48.94 - 38.89 = 10.05\%$  ( $q_{NP}(d) \in [48.94; 38.89]$ ).

To note, that at  $v = 0.1$  and  $d \in [0.05; 0.10]$  occur  $q_{NP}(v) = 0$  (the generated  $CF_t$  values varies too little to imply different solutions) and  $q_{NPE}(v) = 100$  (the influence of CRF values, caused by the considerable difference between  $D_1$  and  $D_2$  values, on



the contrary, is always sufficient to imply different solutions).

**The group of alternatives 6 - dependence on  $d^+$**  (on  $d$  when  $D_1, D_2, I_1$  and  $I_2$  are generated randomly). Initial data:  $d = \{0.05, 0.06, 0.07, \dots, 0.14\}$ ;  $D_2 \in [1; 9]$ ,  $D_1 \in [D_2+1; 10]$ ;  $I_1 \in [100; 1000]$ ,  $I_2 \in [100; 1000]$ ;  $r = 0.2$ ;  $v = 0.5$ . The dependences  $q_{NP}(d^+)$ ,  $q_{NR}(d^+)$ ,  $q_{PR}(d^+)$ ,  $q_{NPE}(d^+)$ ,  $q_{NRE}(d^+)$  and  $q_{PRE}(d^+)$  are shown in Figure 7.



**Figure 7 – Percentages  $q_{NP}(d^+)$ ,  $q_{NR}(d^+)$ ,  $q_{PR}(d^+)$ ,  $q_{NPE}(d^+)$ ,  $q_{NRE}(d^+)$  and  $q_{PRE}(d^+)$**

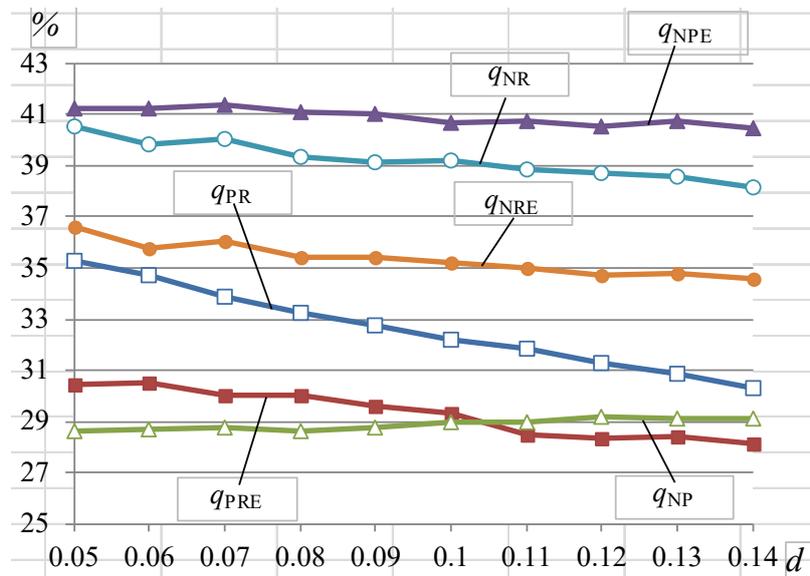
Unlike the group of alternatives 1 (dependence on  $d$ ), for group 6 all six dependences, including the  $q_{NP}(d^+)$ , are decreasing on  $d$ . By pairs, for small values of  $v$  the largest discrepancy is between  $q_{NP}(d^+)$  and  $q_{NPE}(d^+)$ , and for large values of  $v$  the largest discrepancy is between  $q_{PR}(d^+)$ ,  $q_{PRE}(d^+)$ ; the lowest discrepancy usually is between percentages  $q_{NR}(d^+)$  and  $q_{NRE}(d^+)$ . At the same time, one has  $q_{NP}(d^+) < q_{NPE}(d^+)$  and  $q_{PR}(d^+) < q_{PRE}(d^+)$ , but  $q_{NR}(d^+) > q_{NRE}(d^+)$ . The obtained ranges of values for the six dependences are specified in Table 6.

**Table 6 – The ranges of values for the six dependences on  $d^+$  at  $d \in [0.05; 0.14]$**

	$q_{NP}(d^+)$	$q_{NR}(d^+)$	$q_{PR}(d^+)$	$q_{NPE}(d^+)$	$q_{NRE}(d^+)$	$q_{PRE}(d^+)$
Minimum of $q(d^+)$	21.95	24.79	11.31	31.46	20.36	25.53
Maximum of $q(d^+)$	28.03	34.11	23.19	37.47	28.55	28.54

On average, there are a significant number of cases when the use of investigated pairs of indices leads to different solutions; for example  $q_{NPE}(d^+) \in [31.46; 37.47]\%$ . The largest range is that of  $q_{PR}(d^+)$  equal to  $23.19 - 11.31 = 11.88\%$  ( $q_{PR}(d^+) \in [23.19; 11.31]\%$ ), and the narrowest range is that of  $q_{PRE}(d^+)$  equal to  $28.54 - 25.53 = 3.01\%$  ( $q_{PRE}(d^+) \in [28.54; 25.53]$ ).

**The group of alternatives 7 – general group** (on  $d$  when  $D_1, D_2, I_1, I_2, r$  and  $v$  are generated randomly). Initial data:  $d = \{0.05, 0.06, 0.07, \dots, 0.14\}$ ;  $D_2 \in [1; 9]$ ,  $D_1 \in [D_2+1; 10]$ ;  $I_1 \in [100; 1000]$ ,  $I_2 \in [100; 1000]$ ;  $r \in [0.1; 1.0]$ ;  $v \in [0.1; 0.9]$ . The obtained dependences  $q_{NP}(d^+)$ ,  $q_{NR}(d^+)$ ,  $q_{PR}(d^+)$ ,  $q_{NPE}(d^+)$ ,  $q_{NRE}(d^+)$  and  $q_{PRE}(d^+)$  are shown in Figure 8.



**Figure 8 – Percentages  $q_{NP}(d)$ ,  $q_{NR}(d)$ ,  $q_{PR}(d)$ ,  $q_{NPE}(d)$ ,  $q_{NRE}(d)$  and  $q_{PRE}(d)$**

Like the group of alternatives 6 (dependence on  $d^+$ ), for group 7 all six dependences are decreasing on  $d$ . By pairs, the largest discrepancy is between  $q_{NP}(d)$  and  $q_{NPE}(d)$ ; follows, in most cases, the pair  $\{q_{NR}(d), q_{NRE}(d)\}$ , and the lowest discrepancy usually is between frequencies  $q_{PR}(d)$  and  $q_{PRE}(d)$ . At the same time, one has  $q_{NP}(d) < q_{NPE}(d)$ , but  $q_{PR}(d) > q_{PRE}(d)$  and  $q_{NR}(d) > q_{NRE}(d)$ .

The obtained ranges of values for the six dependences are presented in Table 7.

**Table 7 – The ranges of values for the six dependences on  $d$  at  $d \in [0.05; 0.14]$**

	$q_{NP}(d)$	$q_{NR}(d)$	$q_{PR}(d)$	$q_{NPE}(d)$	$q_{NRE}(d)$	$q_{PRE}(d)$
Minimum of $q(d)$	28.59	38.14	30.30	40.47	34.59	28.11
Maximum of $q(d)$	29.17	40.56	35.26	41.38	36.58	30.51

On average, for the group of alternatives 7, the number of cases when the use of researched pairs of indices leads to different solutions is less than 41.38%. The largest range is that of  $q_{PR}(d)$  equal to  $35.26 - 30.30 = 4.96\%$  ( $q_{PR}(d) \in [35.26; 30.30]\%$ ), and the narrowest range is that of  $q_{NP}(d)$  equal to  $29.17 - 28.59 = 0.58\%$  ( $q_{NP}(d) \in [29.17; 28.59]$ ).

**6.4. Generalization of the results of computer simulation**

Dependencies  $q_{NP}(\cdot)$ ,  $q_{NR}(\cdot)$ ,  $q_{PR}(\cdot)$ ,  $q_{NPE}(d)$ ,  $q_{NRE}(d)$  and  $q_{PRE}(\cdot)$  on  $d$  (Figure 2), on  $D_2$  (Figure 3), on  $I_2$  (Figure 4), on  $r$  (Figure 5), on  $v$  (Figure 6), on  $d^+$  (Figure 7) and on  $d$  (Figure 8) are decreasing or slightly decreasing, except that:

- a)  $q_{NP}(d)$ ,  $q_{NP}(v)$ ,  $q_{NP}(d^+)$ ,  $q_{PR}(r)$ ,  $q_{NRE}(D_2)$  and  $q_{NRE}(r)$  are increasing;
- b)  $q_{NPE}(D_2)$ ,  $q_{NPE}(r)$ ,  $q_{PRE}(D_2)$  and  $q_{PRE}(r)$  initially are increasing and after are decreasing, but  $q_{NP}(D_2)$  initially are decreasing and after are increasing;
- c)  $q_{NR}(r)$ ,  $q_{NR}(v)$ ,  $q_{PR}(D_2)$  and  $q_{PRE}(I_2)$  are, practically, invariable.

So, from the total of  $6 \times 7 = 42$  dependences, 21 are decreasing, 6 are increasing, 4 initially are increasing and after are decreasing, 1 initially is decreasing and after is decreasing, and 4 are, practically, invariable.



By pairs, in all seven groups of alternatives of initial data, the largest discrepancy is between  $q_{NP}(\cdot)$  and  $q_{NPE}(\cdot)$  (except the group 6 at large values of  $d$  when this is the pair  $\{q_{PR}(d^+), q_{PRE}(d^+)\}$ ); follows, in most cases, the pair  $\{q_{PR}(\cdot), q_{PRE}(\cdot)\}$ , and the lowest discrepancy usually is between percentages  $q_{NR}(\cdot)$  and  $q_{NRE}(\cdot)$ . At the same time, they take place  $q_{NP}(\cdot) < q_{NPE}(\cdot)$  and  $q_{NR}(\cdot) \geq q_{NRE}(\cdot)$ ; also occurs  $q_{PR}(\cdot) \leq q_{PRE}(\cdot)$  for some groups and  $q_{PR}(\cdot) \geq q_{PRE}(\cdot)$  for other groups.

Thus, the use of EANPV and EAPI indices to compare projects with unequal lives not only allows a more accurate estimation of projects efficiency, but also the solutions obtained may differ more frequently than when using NPV and PI indices. Usually, this statement is also valid for the pairs of indices  $\{EAPI, IRR\}$  and  $\{PI, IRR\}$ , but is an inverse one for the pairs of indices  $\{EANPV, IRR\}$  and  $\{NPV, IRR\}$ .

A comparative analysis of the range of values for the six percentages can be done based on data of Table 8.

**Table 8 – Characteristics of the range of values for the six dependencies, %**

		$q_{NP}(\cdot)$	$q_{NR}(\cdot)$	$q_{PR}(\cdot)$	$q_{NPE}(\cdot)$	$q_{NRE}(\cdot)$	$q_{PRE}(\cdot)$
Minimum of	$q(d)$	7.37	44.25	20.81	71.59	31.66	39.93
	$q(D_2)$	3.38	31.40	4.55	13.18	4.61	9.38
	$q(I_2)$	0	20.40	20.40	43.25	3.89	40.11
	$q(r)$	0.004	26.99	9.66	36.93	15.54	19.40
	$q(v)$	0	38.89	16.31	59.85	25.86	33.99
	$q(d^+)$	21.95	24.79	11.31	31.46	20.36	25.53
	$q(d\cdot)$	28.59	38.14	30.30	40.47	34.59	28.11
Overall minimum		0	20.40	4.55	13.18	4.61	9.38
Maximum of	$q(d)$	23.44	48.39	40.54	93.67	43.72	50.05
	$q(D_2)$	37.50	82.73	75.61	93.61	47.26	50.75
	$q(I_2)$	37.10	57.85	40.54	99.27	56.58	50.38
	$q(r)$	23.34	49.23	49.11	93.76	49.21	50.40
	$q(v)$	22.97	48.94	47.43	100	47.43	53.26
	$q(d^+)$	28.03	34.11	23.19	37.47	28.55	28.54
	$q(d\cdot)$	29.17	40.56	35.26	41.38	36.58	30.51
Overall maximum		37.50	82.73	75.61	100	56.58	50.75
Overall range value		37.50	62.33	71.06	82.82	51.97	41.37

Data of Table 8 show that, at used 7 groups of alternatives of initial data, the average percentage of cases with different solutions for all six pairs of indices usually is considerable, namely:  $q_{NP}(\cdot) \in [0; 37.50]$  %,  $q_{PRE}(\cdot) \in [9.38; 50.75]$  %,  $q_{NRE}(\cdot) \in [4.61; 56.58]$  %,  $q_{NR}(\cdot) \in [20.40; 82.73]$  %,  $q_{PR}(\cdot) \in [4.55; 75.61]$  % and  $q_{NPE}(\cdot) \in [13.18; 100]$  %. Also, the overall size of the value range is approx.: 38 % for  $q_{NP}(\cdot)$ , 41 % for  $q_{PRE}(\cdot)$ , 52 % for  $q_{NRE}(\cdot)$ , 62 % for  $q_{NR}(\cdot)$ , 71 % for  $q_{PR}(\cdot)$  and 83 % for  $q_{NPE}(\cdot)$ .

At the same time, if to consider the uniform distribution of  $q(\cdot)$  in the range interval, the average percentage of cases with different solutions by pairs of indices is approx. (in the increasing order): 18.3 % for  $q_{NP}(\cdot)$ , 30.1 % for  $q_{PRE}(\cdot)$ , 30.6 % for  $q_{NRE}(\cdot)$ , 40.1 % for  $q_{PR}(\cdot)$ , 51.6 % for  $q_{NR}(\cdot)$  and 56.6 % for  $q_{NPE}(\cdot)$ .



## 7. Conclusions

To research comparatively by computer simulation the NPV, PI, EANPV, EAPI and IRR indices, used when selecting investment projects in computerization, the respective general problem is formulated, a model of comparative analysis of projects with unequal lives is defined and the SIMINV application is made up.

Each of the two compared projects is characterized by: discount rate  $d$ , duration  $D$ , volume of investment  $I$  and cash flows  $CF_t$ ,  $t = 1, 2, \dots, D$ . From these characteristics, only the value of  $d$  is common for both projects. The other characteristics in some cases have fixed value and in other cases are generated randomly, in such way forming seven groups of alternatives of initial data.

By computer simulation, the percentage of cases when the solutions, obtained using indices of each of the pairs  $\{\text{NPV, PI}\} - q_{\text{NP}}$ ,  $\{\text{NPV, IRR}\} - q_{\text{NR}}$ ,  $\{\text{PI, IRR}\} - q_{\text{PR}}$ ,  $\{\text{EANPV, EAPI}\} - q_{\text{NPE}}$ ,  $\{\text{EANPV, IRR}\} - q_{\text{NRE}}$  and  $\{\text{EAPI, IRR}\} - q_{\text{PRE}}$ , differ is determined. These results complement, to some extent, the known theoretical ones in the domain. Thus, for all seven groups of alternatives of initial data are determined:

- the character of dependencies  $q_{\text{NP}}(\cdot)$ ,  $q_{\text{NR}}(\cdot)$ ,  $q_{\text{PR}}(\cdot)$ ,  $q_{\text{NPE}}(\cdot)$ ,  $q_{\text{NRE}}(\cdot)$  and  $q_{\text{PRE}}(\cdot)$ ;
- the relation larger/smaller between percentages of the pairs  $\{q_{\text{NP}}(\cdot), q_{\text{NPE}}(\cdot)\}$ ,  $\{q_{\text{NR}}(\cdot), q_{\text{NRE}}(\cdot)\}$  and  $\{q_{\text{PR}}(\cdot), q_{\text{PRE}}(\cdot)\}$ . For example, the use of EANPV and EAPI indices to compare projects with unequal lives not only allows a more accurate estimation of projects efficiency, but also the solutions obtained may differ more frequently than when using NPV and PI indices. Usually, this statement is also valid for the pairs of indices  $\{\text{EAPI, IRR}\}$  and  $\{\text{PI, IRR}\}$ , but is an inverse one for the pairs of indices  $\{\text{EANPV, IRR}\}$  and  $\{\text{NPV, IRR}\}$ ;
- the average percentage of cases with different solutions by pairs of indices is approx. (in the increasing order): 18.3 % for  $q_{\text{NP}}(\cdot)$ , 30.1 % for  $q_{\text{PRE}}(\cdot)$ , 30.6 % for  $q_{\text{NRE}}(\cdot)$ , 40.1 % for  $q_{\text{PR}}(\cdot)$ , 51.6 % for  $q_{\text{NR}}(\cdot)$  and 56.6 % for  $q_{\text{NPE}}(\cdot)$ .

Thus, the average percentage of cases with different solutions is considerable; it depends on the used pair of indices, but usually overpasses 18 %, if the EAV method is not used, and overpasses 30 %, if the EAV method is used.

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**Аннотация.** Теоретические результаты не всегда дают однозначный ответ относительно предпочтительности использования показателей эффективности инвестиций в ИТ-проекты. Чтобы дополнить некоторые из таких результатов, индексы чистой приведенной стоимости (NPV), прибыльности (PI), эквивалентной годовой NPV (EANPV), эквивалентной годовой PI (EAPI) и внутренней нормы доходности (IRR) исследуются компьютерным моделированием. Для этого сформулирована соответствующая общая задача, разработана модель сравнительного анализа проектов с неравным сроком службы и составлено приложение SIMINV. Используя SIMINV, процент случаев, когда решения, полученные с использованием показателей каждой из пар  $\{NPV, PI\} - NP$ ,  $\{NPV, IRR\} - NR$ ,  $\{PI, IRR\} - PR$ ,  $\{EANPV, EAPI\} - NPE$ ,  $\{EANPV, IRR\} - NRE$  и  $\{EAPI, IRR\} - PRE$  различаются, для семи альтернатив групп исходных данных. На основании проведенных расчетов были выявлены некоторые свойства показателей, в том числе: характер зависимости от исходных данных; отношение больше/меньше между процентами каждой из пар  $\{NP, NPE\}$ ,  $\{NR, NRE\}$  и  $\{PR, PRE\}$  (например, использование индексов EANPV и EAPI для сравнения проектов с неравным сроком службы позволяет не только более точно оценить эффективность проектов, но и полученные решения могут отличаться чаще, чем при использовании показателей NPV и PI); общий размер диапазона значений и максимальный средний процент случаев с различными решениями, который составляет ок. 57% для пары индексов EANPV и EAPI и от 18% до 52% для остальных пяти пар индексов.

**Ключевые слова:** инвестиционные проекты, сравнительный анализ, чистая приведенная стоимость, индекс рентабельности, внутренняя норма доходности, метод эквивалентной годовой стоимости, компьютерное моделирование.

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