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**WAVELET ANALYSIS OF THERMAL WAVE PROPAGATION IN
NON-UNIFORM REINFORCED COMPOSITES**
**ВЕЙВЛЕТ-АНАЛІЗ ПОШИРЕННЯ ТЕПЛОВИХ ХВИЛЬ У НЕОДНОРІДНИХ
АРМОВАНИХ КОМПЗИТАХ**

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Abstract. *The goal of this work was to build a software model that allows one to detect the local location of inclusions in the volume of a reinforced composite. The numerical model included blocks for statistical processing of synchronous infrared thermography data as a non-destructive testing method, as well as wavelet transformations of basic functions describing the propagation of thermal waves in a composite material. Software based on the finite difference method was developed for the simulation. To study the thermal response, the effects of applied heat flux and time were analyzed for inclusions of various sizes and depths. The calculations used the wavelet transform to calculate phase angle data based on the temperature-time history of each local inclusion. The influence of such wavelet parameters as scale, shift, and modulation frequency on the results of the analysis of thermal wave propagation was analyzed.*

Key words: *thermal waves, wavelet transformations, composite structures, inhomogeneous field.*

Introduction.

Currently, composite structures, due to their outstanding thermal properties, as well as high strength and corrosion resistance, are used in a wide range of industries, including: aerospace, automotive, transportation, agriculture, construction, civil infrastructure, hostile environments, electric vehicle charging and others [1 - 3].

However, micro-inclusions and other violations of the homogeneity of the material can pose a fairly serious threat to composite structures [4, 5]. This implies the need for constant monitoring of the composite structure both during the manufacturing process and throughout the entire period of operation. In practice, several thermographic techniques are common as remote analysis of the structure of a composite material.

One of the most common techniques is infrared thermography [6, 7], which is used as a non-destructive testing method to detect defects in laminar composite materials. The advantage of this method is the ability to scan over a wide range, as well as the possibility of contactless and quick control. In addition, interpretation of thermal data can be accomplished using simple computational techniques.

Synchronous thermography [8, 9] is also one of the most commonly used thermographic methods for non-destructive examination of composite structures. The synchronous thermography method involves excitation of the sample with a heat flux, which depends on time according to the sine law. The temperature of the surface of the composite through which the thermal wave passes is controlled using a thermal receiver, for example, an infrared camera. The field of inhomogeneity of the composite



material, in particular, the internal structure and properties of the material itself are the determining factors that affect the heat transfer of the surface. Therefore, by developing appropriate statistical methods for processing monitoring data, a dynamic profile of the amplitude and phase angle of the thermal wave can be constructed. The module of the absolute difference between the phase angle of the defect and the phase angle of the sound regions makes it possible to construct a dynamic profile of the phase angle, which, in turn, provides the necessary conditions for measuring the phase contrast.

The final stage of the synchronous thermography technique is to determine the properties of the material and near-surface defects based on the characteristic features of the phase contrast profile. It should be noted that plotting the dependence of the depth of inclusions on the surface coordinate of a composite sample can also be determined from the phase contrast profile.

This work presents the modeling and analysis of the results of the detection process of inclusions in a reinforced composite specimen using a synchronous thermography method. The software implementation of the process of propagation of thermal waves in the volume of a composite material was based on the finite difference method. The main objective of the study was to simulate the process of lock-in thermographic detection of inclusions in reinforced composite material. The second goal was to compare the results of simulations and numerical experiments processed using the wavelet transform. Thus, another goal was to study the influence of wavelet parameters such as scale and shift on the ability to determine the size and depth of inclusions. In addition, the influence of wavelet transform modes on the modulation frequency of the phase contrast profile was determined.

Wavelet transform of composite thermographic data.

The analysis carried out in this work is based on the assumption that the phase image profile is independent of local perturbations such as non-uniform heating and emissivity. In addition, the phase image profile is primarily affected by the depth value, which corresponds to the passage of a thermal wave in the composite sample. These trends suggest that phase imaging can be used to evaluate inclusions in a composite material. Numerical data related to phase angle and amplitude can be obtained from the time profile of the temperature of each elementary region of the composite volume using the Fourier transform and wavelet transform, which particularly distinguishes these methods from other data processing methods. A necessary condition for the possibility of statistical processing of a set of thermal images for the purpose of detecting and characterizing defects is the numerical analysis of the transient signal.

However, the Fourier transform uses a series of infinite sine waves to recovery of any signal. Therefore, this way of representing a function can cause some problems when the signal contains transient components and/or abrupt changes. The way out of this situation is to use the wavelet transform, namely an extension of the Fourier transform, which preserves time information. This information directly correlates with the depth of the defect, while retaining the interesting features of the Fourier transform. The final stage of the wavelet transform allows us to obtain a set of time-frequency representations of the signal with different resolutions. Ultimately, it can be argued that the wavelet transform is an alternative processing algorithm that allows one to obtain a better representation of transition functions.



The thermographic method for describing the thermodynamic characteristics of a homogeneous reinforced composite material involves the use of the Fourier equation in the following form

$$\rho c \frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2} + \rho H_r \frac{\partial \alpha}{\partial t} \quad (1)$$

$$\frac{\partial \alpha}{\partial t} = F(\alpha, t), \quad (2)$$

where T is the temperature; ρ , C_p and κ are the density, specific heat, and thermal conductivity of the composite, respectively; $\partial \alpha / \partial t$ is the speed of heat passing through defects; H_r is the total heat of reaction; $F(\alpha, t)$ represents kinetic characterization.

In this work $F(\alpha, t)$ is modeled as suggested by

$$F(\alpha, t) = K(T) \alpha^m (1 - \alpha)^n, \quad (3)$$

where α is degree of heat passing through composite defects; m and n describe the order of the heat process and K follows the Arrhenius law

$$K = A \exp\left(-\frac{E}{RT}\right), \quad (4)$$

where E is the heat energy; R is the gas constant.

The initial and the boundary conditions are as follows

$$T(x, 0) = T_0; \quad \alpha(x, 0) = 0, \quad -l \leq x \leq l \quad (5)$$

$$T(\pm l, t) = T_C; \quad 0 \leq t \leq t_f. \quad (6)$$

In the inverse problem considered in this study, the kinetic parameters (m , n , A , E) are unknown and must be estimated from the measured temperatures of several sensors during the passage of the temperature wave. Before applying the thermographic data processing method, the averaged values of the kinetic parameter and measured temperatures are considered. This defines the sum of squared error function that should be minimized because it is commonly used in inverse methods:

$$S = \sum_{j=1}^N \sum_{i=1}^M (T_{i,j} - Y_{i,j})^2 = U^T U, \quad (7)$$

where $Y_{i,j}$ are the temperature values before the thermographic treatment procedure; $T_{i,j}$ are the temperature values after the thermographic treatment procedure.

The value M is the number of total time steps, and N is the number of the used sensors. The function S is a function of the unknown kinetic parameters (m , n , A , E).

According to the wavelet transform principle, the analyzed function $f(t)$ is decomposed into a set of basic functions, which are referred to as wavelets. The results of the synchronous thermography technique are compared with data from the continuous wavelet transform

$$\beta(s, \tau) = \int_{-\infty}^{\infty} f(t) \Psi_{s, \tau}^*(t) dt = \text{Re}_n + j \text{Im}_n, \quad (8)$$

where symbol $*$ is used for complex conjugation; s is the scale and defines the wavelet dilation ($s > 1$) or contraction ($1 > s > 0$); τ corresponds to the wavelet translation along



the analyzed signal; t is the time; Re_n is the real part of the transform, Im_n is the imaginary part of the transform; j is the imaginary number.

Function is generated by scaling and translating the base wavelet

$$\beta_{s,\tau}(t) = \frac{1}{\sqrt{s}} \Psi\left(\frac{t-\tau}{s}\right). \quad (9)$$

In this study Morlet wavelet is used as base wavelet

$$\Psi(x) = \frac{1}{\sqrt{\pi f_b}} \exp(2\pi j f_c x) \cdot \exp\left(-\frac{x^2}{f_b}\right), \quad (10)$$

where f_b is a constant that defines the width of wavelet; f_c is the wavelet center frequency constant. In this work, $f_b = 1$ and $f_c = 1$ were considered.

The phase angle profile during the passage of a temperature wave through the volume of a reinforced composite sample can be determined according to the following equation

$$\varphi_n = \frac{1}{\tan\left(\frac{Im_n}{Re_n}\right)}. \quad (11)$$

The study of thermographic data was carried out under the assumption that the source of thermal excitation generated a thermal flux, the characteristics of which changed over time according to a harmonic law

$$Q = \frac{Q_0}{2} [1 + \cos(2\pi f t)], \quad (12)$$

where Q is the incident heat flux power density; Q_0 is the intensity of the heating source; f is the modulation frequency; t is the excitation time.

Numerical calculations were carried out for the following data set: amplitude source power is 2 kW. The time step was determined by the modulation frequency, and the analysis period was determined by $50f$, where f is the modulation frequency. The simulation was conducted at different excitation frequencies ranging from 2 Hz down to 0.05 Hz.

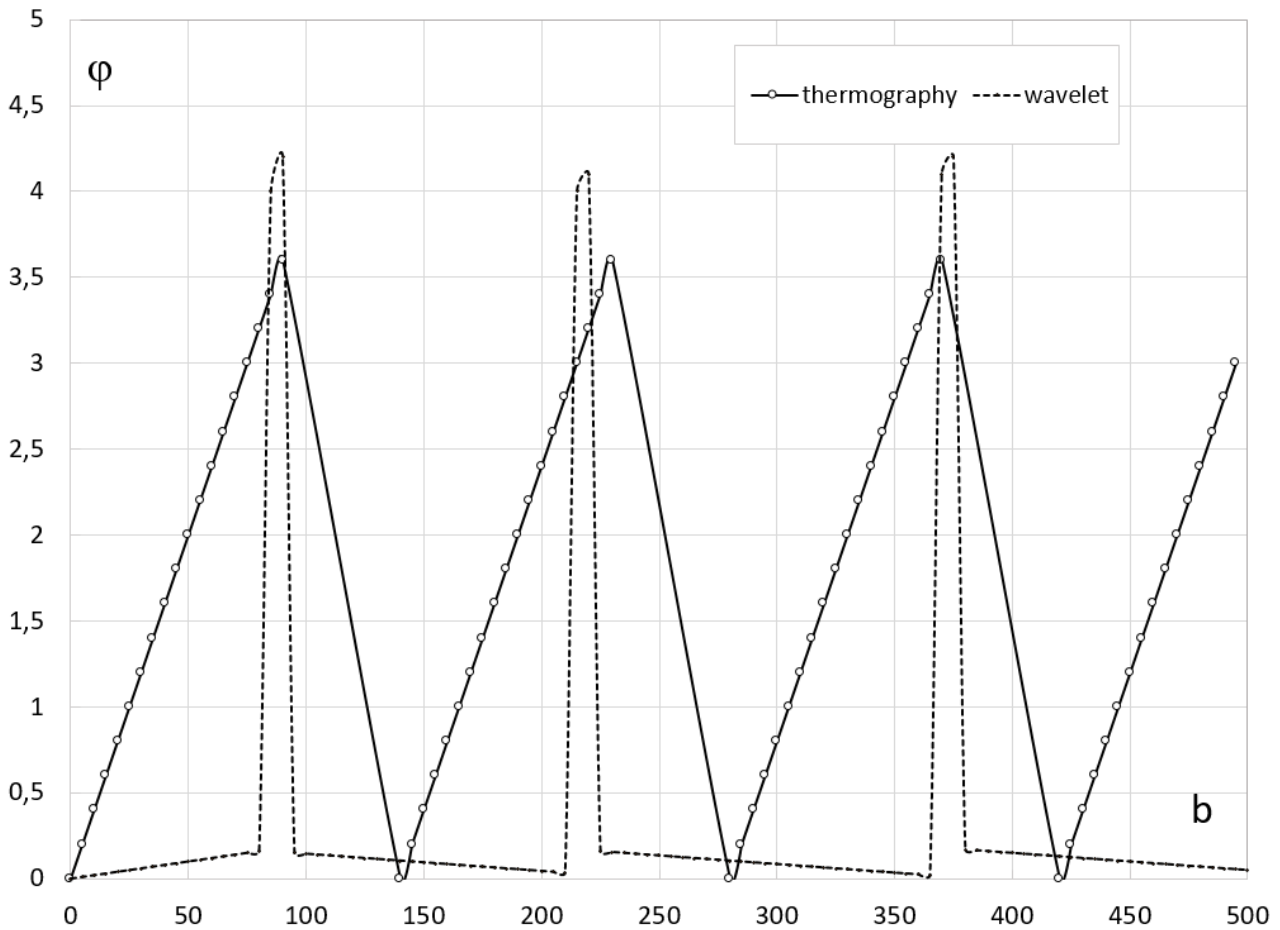
The phase angle profile of thermal waves for various scale factors (from 0.25 to 1) was carried out at modulation frequencies corresponding to the detected inclusions. The calculation of the phase contrast profile was carried out according to the formula

$$\Delta C = C_d - C_s, \quad (13)$$

where C_d is the phase angle of the inclusion; C_s is the phase angle of the fixed heat wave.

The calculation results are shown in Figures 1 and 2. The maximum shifts for the phase angle profile calculated using the synchronous thermography and wavelet transform methods are in satisfactory agreement.

The relative difference in the depth of location under the surface of the composite material for penetrations A and B was 1.4 times. It should be noted that the phase angle decreases to zero approximation value with a significant increase in the carrier frequency to 2 Hz.



**Figure 1 – Phase angle profile of inclusion A:
φ - phase contrast, b - shifts.**

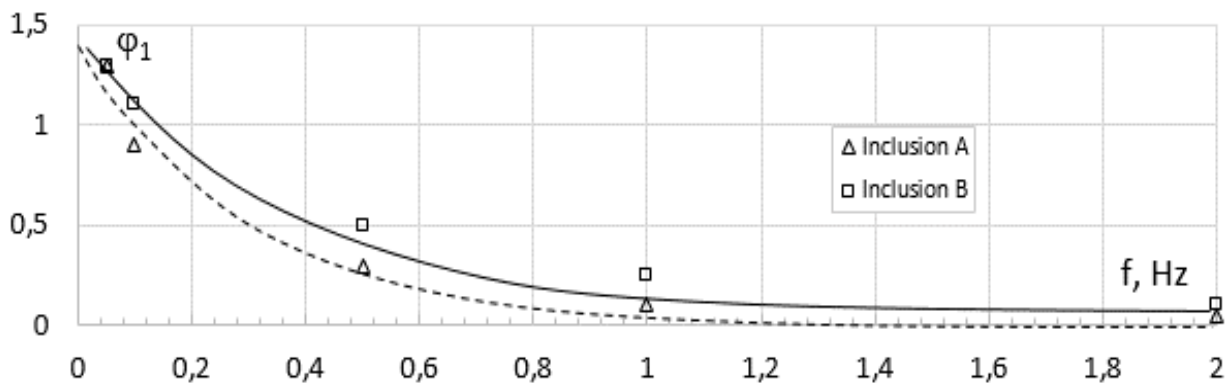


Figure 2 - Phase profile obtained by Wavelet transform for inclusions A and B.

Conclusions.

The calculation results show that the wavelet transform method has proven to be effective in post-processing thermal images in thermography. The numerical characteristics of the phase profile depend on the modulation frequency and the choice of wavelet parameters, in particular, on the scale and shift. The phase contrast turned out to be maximum at the optimal modulation frequency of 0.2 Hz. For fixed sizes of inclusions in the composite material, the phase contrast can increase or decrease.



It was observed that the phase contrast increased with increasing inclusion depth at a modulation frequency of 0.2 Hz. In addition, it was found that at a fixed depth, the phase contrast increased as the size of the inclusion decreased.

References:

1. Mangino E., Carruthers J., and Pitarresi G. The future use of structural composite materials in the automotive industry. *International Journal of Vehicle Design*, 2007. – V. 44(3-4). – Pp. 211-232. DOI: 10.1504/IJVD.2007.013640.
2. Awad Z.K., Aravinthan T., Zhuge Y., and Gonzalez F. A review of optimization techniques used in the design of fibre composite structures for civil engineering applications. *Materials & Design*, 2012. – V. 33. – Pp. 534-544. DOI: 10.1016/j.matdes.2011.04.061.
3. Hsissou R., Seghiri R., Benzekri Z., Hilali M., Rafik M., and Elharfi A. Polymer materials: A comprehensive review. *Composite Structures*, 2021. – V. 262. – 113640. DOI: 10.1016/j.compstruct.2021.113640.
4. Durão, L.M.P. et al. Experimental Study of Drilling Damage Outcomes in Hybrid Composites with Waste Micro-Inclusions. *Materials*, 2023. – V. 16. – 7325. DOI:10.3390/ma16237325.
5. Ogierman W., and Kokot G. Generation of the representative volume elements of composite materials with misaligned inclusions. *Composite Structures*, 2018. – V. 201. – Pp. 636-646. DOI: 10.1016/j.compstruct.2018.06.086.
6. Meolo C., Baccardi S., Carlomagno G.M., Boffa N.D., Monaco E., and Ricci F. Nondestructive evaluation of carbon fibre reinforced composites with infrared thermography and ultrasonics. *Composite Structures*, 2015. – V. 134(15). – Pp. 845-853. DOI: 10.1016/j.compstruct.2015.08.119.
7. Li Y., Yang Z., Zhu J., Ming A., Zhang W., and Zhang J. Investigation on the damage evolution in the impacted composite material based on active infrared thermography. *NDT & E International*, 2016. – V. 83. – Pp. 114-122. DOI: 10.1016/j.ndteint.2016.06.008.
8. Louaayou M., Naït-Saïd N., and Louai F.Z. 2D finite element method study of the simulation induction heating in synchronic thermography NDT. *NDT & E International*, 2008. – V. 41(8). – Pp. 577-581. DOI: 10.1016/j.ndteint.2008.07.001.
9. Biju N., Ganesan N., Krishnamurthy C.V., and Balasubramaniam K. Defect sizing simulation studies for the tone – burst eddy current thermography using genetic algorithm based inversion. *Journal of Nondestructive Evaluation*, 2012. – V. 31. – Pp. 342-348. DOI: 10.1007/s10921-012-0143-y.

Анотація. Метою даної роботи було побудувати програмну модель, яка дозволяє виявити локальне розташування включень в об'ємі армованого композиту. Чисельна модель включала блоки статистичної обробки даних синхронної інфрачервоної термографії як методу неруйнівного контролю, а також вейвлет-перетворення основних функцій, що описують поширення теплових хвиль у композитному матеріалі. Для моделювання було розроблено програмне забезпечення на основі методу кінцевих різниць. Щоб вивчити тепловий відгук, вплив прикладеного теплового потоку та часу було проаналізовано для включень різного розміру та глибини. У розрахунках використовувалося вейвлет-перетворення для обчислення даних фазового кута на основі температурно-часової історії кожного локального



включення. Проаналізовано вплив таких параметрів вейвлету, як масштаб, зсув і частота модуляції, на результати аналізу поширення теплової хвилі.

Ключові слова: теплові хвилі, вейвлет-перетворення, композитні структури, неоднорідне поле.

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