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# PARAMETRIC STUDY OF FUNCTIONALLY GRADED MATERIALS USING THE HAAR WAVELET METHOD

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***Abstract.*** *This paper studies the effect of frequency-dependent damping of composite materials on the kinematic characteristics of rotating laminated composite cylindrical shells. Based on the Haar wavelet method, the dynamic equations and corresponding boundary conditions were discredited, and a standard equation for eigenvalues was obtained. The dispersion characteristics of the attenuation parameters of composite materials were determined using the complex moduli method. The eigenvalue equation is solved by iterative calculation to obtain the modal frequency and damping characteristics of rotating composite cylindrical shells. The influence of frequency, rotation speed, lamination and geometric parameters on the damping properties of a laminated composite shell is determined for four typical boundary conditions. The calculation results indicate the need to take into account the frequency dependence of attenuation of composite materials, especially when analyzing low-order modes. Numerical experiments show the presence of direct and reverse attenuation, corresponding to the modes of direct traveling wave and reverse traveling wave, and the motion of the direct wave is more stable. It was found that certain characteristics of rotation and self-damping of the material can lead to loss of rotational stability of the composite cylindrical shell.*

***Key words:*** *composite shells, Haar wavelet method, free vibrations, boundary condition.*

## Introduction.

The widespread use of multilayer composite structures in various applications is due to their specific properties, namely high stiffness-to-weight and strength-to-weight ratios [1, 2]. On the other hand, some undesirable properties (typical of multilayer composite structures), i.e. transverse shear deformability and transverse anisotropy, must be accurately described in order to correctly predict their structural response. A large number of studies have been devoted to vibration analysis of cylindrical shells, aimed at providing an understanding of the dynamic behavior, optimal design, and avoidance of unpleasant, inefficient, and structurally damaging resonance of complex composite cylindrical shells [3].

Due to the mechanical complexity of shell structures, various shell theories have been proposed and extensive research has been conducted based on these theories. It was of great interest and technical importance for researchers to develop an accurate and efficient method that could be used to determine the vibration characteristics (natural frequencies, modes, etc.) of composite layered cylindrical shells.

One of the promising approaches is the use of the Haar wavelet to analyze free vibrations of composite laminated cylindrical shells [4]. Current wavelet-based approaches include wavelet collocation, wavelet finite elements, etc. In most wavelet methods, calculating the coupling coefficients of wavelets is a complex task [5, 6]. It is obvious that attempts to simplify solutions based on wavelet methods are required.

Recently, sufficient interest has been paid to the Haar wavelet functions, which are the mathematically simplest wavelets.

In this paper, a numerical discretization procedure based on Haar wavelets is applied to the analysis of vibrations of composite layered cylindrical shells subject to various boundary conditions.

It is important to study the dynamic characteristics of composite shell considering the internal damping of the material for the optimal design and improvement of the service life of the structure. Damping is an important parameter of fiber-reinforced composite materials, and accurate prediction of the damping characteristics of composite structures is the basis for identifying the behavior of the dynamic response under various loads [7, 8].

As frequently used methods, the complex modulus method and the strain energy method can be used to predict the damping characteristics of composite structures. This method involves the procedure of expressing the elastic modulus of the material in complex form, where the damping loss factor is the ratio of the imaginary part to the real part of the complex modulus.

The mechanical properties of typical viscoelastic materials, which include composite materials, are obviously frequency dependent. The traditional complex constant modulus model cannot adequately describe the mechanical properties of materials, which will lead to overestimation of predicted attenuation values [9, 10].

It is necessary to take into account that cylindrical multilayer composite shells sometimes operate in a rotational state, which is usually accompanied by a process of increasing speed. Since the viscoelasticity of composite materials will affect the stability of the rotating cylindrical shell during the process of increasing and decreasing speed, it is necessary to study the damping characteristics for this case.

The present study plan uses the shell theory and Hamilton's principle to construct the equation of a rotating cylindrical shell. Then, the modal frequency and damping behavior of the rotating composite cylindrical shell are determined using the Haar wavelet method. Finally, the influence of various aspects including the rotation speed, circumferential wave number, frequency, lamination patterns and geometric parameters on the damping properties of the composite cylindrical shells under different boundary conditions is investigated.

**Equations of motion and boundary conditions.**

Let us consider the movement of a cylindrical shell consisting of a multilayer composite. The motion of the structure can be reduced to rotation around its axis with a constant angular velocity Ω, length *L* and thickness *h*. The radius of the middle surface of the cylindrical shell is *R*. An orthogonal curvilinear coordinate system ($x$, $\theta$, $z$) is fixed on the middle surface of the cylindrical shell, where $u$, $v$ and $w$ represent the displacement in the axial $x$, circumferential $\theta$ and radial $z$ directions, respectively. The number of laminated layers of the composite cylindrical shell is $N$, and the coordinates of the upper and lower surfaces of layer $k$ are represented by $z_k$ and $z_{k-1}$, respectively.

According to the linear theory of thin shells, the strain in any joint depends on the shape strain $\varepsilon_x^0$, $\varepsilon_\theta^0$, $\varepsilon_{x\theta}^0$ and the curvature components $\kappa_x^0$, $\kappa_\theta^0$, $\kappa_{x\theta}^0$ between the planes, the relationship between the strain and the stability displacement, as

$$\varepsilon_x = \varepsilon_x^0 + z\kappa_x^0, \quad \varepsilon_\theta = \varepsilon_\theta^0 + z\kappa_\theta^0, \quad \varepsilon_{x\theta} = \varepsilon_{x\theta}^0 + z\kappa_{x\theta}^0 , \tag{1}$$

where

$$\varepsilon_x^0 = \frac{\partial u}{\partial x}, \quad \varepsilon_\theta^0 = \frac{\partial v}{R\partial\theta} + \frac{w}{R}, \quad \varepsilon_{x\theta}^0 = \frac{\partial v}{\partial x} + \frac{\partial u}{R\partial\theta}, \tag{2}$$

$$\kappa_x^0 = -\frac{\partial^2 w}{\partial x^2}, \quad \kappa_\theta^0 = -\frac{\partial v}{R^2\partial\theta} - \frac{\partial^2 w}{R^2\partial\theta^2}, \quad \kappa_{x\theta}^0 = -\frac{\partial v}{R\partial\theta} - 2\frac{\partial^2 w}{R\partial x\partial\theta}. \tag{3}$$

The mechanical stress at any point of the k-th layer of the shell can be described by the following relationship

$$\Psi(\sigma_x, \sigma_\theta, \sigma_{x\theta}) = F_2\left[\overline{Q}_{ik}\right]\ \Psi(\varepsilon_x, \varepsilon_\theta, \varepsilon_{x\theta}), \tag{4}$$

where $F_2$ is numerically equal to the reduced off-axis stiffness matrix, which satisfies the following transformation relation with the reduced on-axis stiffness matrix $[Q_{ij}]$

$$\begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix} = \mathbf{T} \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \mathbf{T}^T, \quad (5)$$

in this case the transformation matrix **T** can be expressed as

$$T = \begin{bmatrix} \cos^2\beta & \sin^2\beta & -2\cos\beta\sin\beta \\ \sin^2\beta & \cos^2\beta & 2\cos\beta\sin\beta \\ \cos\beta\sin\beta & -\cos\beta\sin\beta & \cos^2\beta-\sin^2\beta \end{bmatrix}. \quad (6)$$

In equation (4), the numerical value $\beta$ can be equated to the angle between the fiber direction and the coordinate direction. The reduced stiffness matrix is characterized by elements $[Q_{ij}]$, which correspond to the following relations

$$Q_{11} = \frac{E_1}{1-\mu_{12}\mu_{21}}, \quad Q_{12} = \frac{\mu_{12}E_2}{1-\mu_{12}\mu_{21}}, \quad Q_{22} = \frac{E_2}{1-\mu_{12}\mu_{21}}, \quad Q_{66} = G_{12}. \quad (7)$$

The internal force $N = \{N_x, N_\theta, N_{x\theta}\}^T$ and internal moment $M = \{M_x, M_\theta, M_{x\theta}\}^T$ of the cylindrical shell are defined as

$$\mathbf{N}^T = \int_{-h/2}^{+h/2} (\sigma_x, \sigma_\theta, \sigma_{x\theta})\,dz, \quad \mathbf{M}^T = \int_{-h/2}^{+h/2} (\sigma_x, \sigma_\theta, \sigma_{x\theta})\,zdz. \quad (8)$$

As a result, the matrix relationship between the internal forces and mechanical stresses in a cylindrical laminated shell, taking into account the previous equations, has the following form

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon^0 \\ \kappa^0 \end{Bmatrix}, \quad (9)$$

in which $\varepsilon^0 = \{\varepsilon_x^0, \varepsilon_x^0, \varepsilon_x^0\}^T$, $\kappa^0 = \{\kappa_x^0, \kappa_x^0, \kappa_x^0\}^T$, and $A$, $B$ and $D$ respectively represent the matrices of tensile stiffness, adhesion stiffness and bending stiffness. The elements of the specified quantities are defined as follows

$$A_{ij} = \sum_{k=1}^{N} \overline{Q}_{ij}(z_k - z_{k-1}), \quad (10)$$

$$B_{ij} = \sum_{k=1}^{N} \overline{Q}_{ij}(z_k^2 - z_{k-1}^2), \quad (11)$$

$$D_{ij} = \sum_{k=1}^{N} \overline{Q}_{ij}\left(z_k^3 - z_{k-1}^3\right),\ i,j = 1,2,6. \tag{12}$$

The strain energy of the composite cylindrical shell can be expressed as

$$U = \frac{1}{2}\int_{-h/2}^{+h/2}\int_{0}^{2\pi}\int_{0}^{L}\left(N_x\varepsilon_x^0 + N_\theta\varepsilon_\theta^0 + N_{x\theta}\varepsilon_{x\theta}^0 + M_x\kappa_x^0 + M_\theta\kappa_\theta^0 + M_{x\theta}\kappa_{x\theta}^0\right)Rdxd\theta\,. \tag{13}$$

In this paper, several types of boundary conditions are considered, namely, fixed at both ends (*A*), fixed at one end and free at the other end (*B*), fixed at one end and simply supported at the other end (*C*), and simply supported at both ends (*D*). The constraint equations corresponding to these four boundary conditions are given by

boundary condition *A*

$$u = 0,\quad v = 0,\quad w = 0,\quad \frac{\partial w}{\partial x} = 0,\quad x = 0, L; \tag{14}$$

boundary condition *B*

$$u = 0,\quad v = 0,\quad w = 0,\quad \frac{\partial w}{\partial x} = 0,\quad x = 0; \tag{15}$$

$$N_x = 0,\quad M_x = 0,\quad N_{x\theta} + \frac{M_{x\theta}}{R} = 0,\quad \frac{\partial M_x}{\partial x} + \frac{2}{R}\frac{\partial M_{x\theta}}{\partial \theta} = 0,\quad x = L; \tag{16}$$

boundary condition *C*

$$u = 0,\quad v = 0,\quad w = 0,\quad \frac{\partial w}{\partial x} = 0,\quad x = 0; \tag{17}$$

$$u = 0,\quad v = 0,\ N_x = 0,\quad M_x = 0,\quad x = L; \tag{18}$$

boundary condition *D*

$$u = 0,\quad v = 0,\ N_x = 0,\quad M_x = 0,\quad x = 0; \tag{19}$$

According to the method of separation of variables and the Haar wavelet discrete space technique, the solution to the displacement of a traveling wave with a circumferential wave number *n* can be represented as follows:

$$u(x,\theta,t) = U(x)\cos(n\theta + \omega t), \tag{20}$$

$$v(x,\theta,t) = V(x)\sin(n\theta + \omega t), \tag{21}$$

$$w(x,\theta,t) = W(x)\cos(n\theta + \omega t), \tag{22}$$

in which $U(x)$, $V(x)$ and $W(x)$ represent, respectively, the forms of axial vibrations in three directions of deformation, the forms of which are associated with the boundary conditions.

The highest order derivatives of the displacements $U$, $V$, and $W$ in the above equation are expressed by the Haar wavelet series

$$U''(\xi)=\sum_{i=1}^{2M} a_i h_i(\xi),\quad V''(\xi)=\sum_{i=1}^{2M} b_i h_i(\xi),\quad W^{IV}(\xi)=\sum_{i=1}^{2M} c_i h_i(\xi), \tag{23}$$

where $a_i$, $b_i$, and $c_i$ denote the unknown wavelet coefficients, and $h_i(\xi)$ is Haar wavelet function. M is the quality defined by $M = 2J$, in which $J$ is the maximal level of resolution.

As a result, the dynamic equation can be transformed into algebraic equations for the eigenvalues in the following form

$$[\omega^2\mathbf{M}+\omega\mathbf{G}+\mathbf{K}]\,\mathbf{X}=0, \tag{24}$$

where **M** and **G** are the mass matrix and the gyroscopic matrix, respectively, and **K** is the stiffness matrix due to elastic deformation and rotation.

The equation of eigenvalues can be reduced to the canonical form

$$\left(\begin{bmatrix} 0 & I \\ -\mathbf{K} & -\mathbf{G} \end{bmatrix} - \omega \begin{bmatrix} \mathbf{I} & 0 \\ 0 & \mathbf{M} \end{bmatrix}\right)\cdot\begin{Bmatrix} \mathbf{X} \\ \omega\mathbf{X}^* \end{Bmatrix}=0. \tag{25}$$

The loss coefficient is proportional to the ratio of the imaginary and real parts of the corresponding frequency

$$\eta = 2\frac{\mathrm{Im}(\omega)}{\mathrm{Re}(\omega)}. \tag{26}$$

Forward and backward attenuation correspond to the forward and backward motion of the traveling wave. The relationship between their values and the stability of the system is as follows: for positive values of the loss factor $\eta > 0$, the system is stable; when $\eta < 0$, the system is unstable; when $\eta = 0$, the system is in a critical state between stability and instability. This model considers the generalized case of composite materials that depend on frequency. Accordingly, it makes sense to use an iterative algorithm to determine the $m$-th order natural frequency ($m$ is also known as the axial half-wave number) and the loss factor for a certain circumferential wave number $n$.

Table 1 illustrates the variation of forward $\omega_f$ and backward $\omega_b$ frequencies of cross-reinforced $[0^0/90^0/0^0]$ rotating composite cylindrical shells with resolution parameter $J$ under different boundary conditions $A$, $B$, $C$ and $D$ ($J$ is the resolution number, $m = 1$, $n = 6$, $\Omega = 0.7$ rev/s).

**Table 1 – Variation of frequency parameters**

| $J$ | $\omega_f$ | | | $\omega_b$ | | |
|---|---|---|---|---|---|---|
| | $A$ | $B$ | $C$ | $B$ | $C$ | $D$ |
| 3 | 0.3527 | 0.3108 | 0.2671 | 0.3128 | 0.2689 | 0.2130 |
| 4 | 0.3515 | 0.3106 | 0.2604 | 0.3127 | 0.2634 | 0.2147 |
| 5 | 0.3522 | 0.3101 | 0.2638 | 0.3154 | 0.2677 | 0.2114 |
| 6 | 0.3541 | 0.3107 | 0.2684 | 0.3162 | 0.2691 | 0.2128 |
| 7 | 0.3578 | 0.3105 | 0.2616 | 0.3191 | 0.2652 | 0.2175 |

Figures 1 and 2 show the variation of modal attenuation and natural frequencies of a rotating composite cylindrical shell depending on the rotation speed for a fixed number of wave numbers and axial waves. These figures represent the forward and backward traveling wave modes, respectively. The presented results indicate that the effect of rotation speed on modal attenuation is not the same for different numbers of axial waves. The modal attenuation of the forward traveling wave initially increases and then decreases with increasing rotation speed, while the modal attenuation of the backward traveling wave always decreases with increasing rotation speed.

The change in damping is opposite to the change in natural frequency with increasing rotation speed, that is, the damping decreases with increasing natural frequency. It can be concluded that the damping property of the composite cylindrical shell will decrease with increasing rigidity.

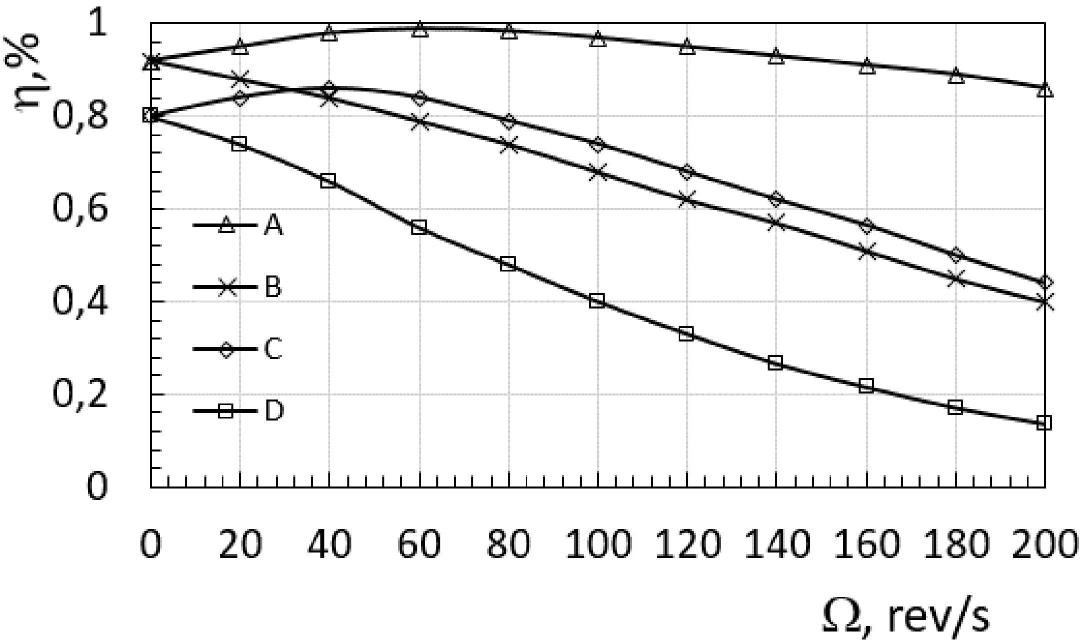


**Figure 1 - Variation of modal damping with rotating speed**

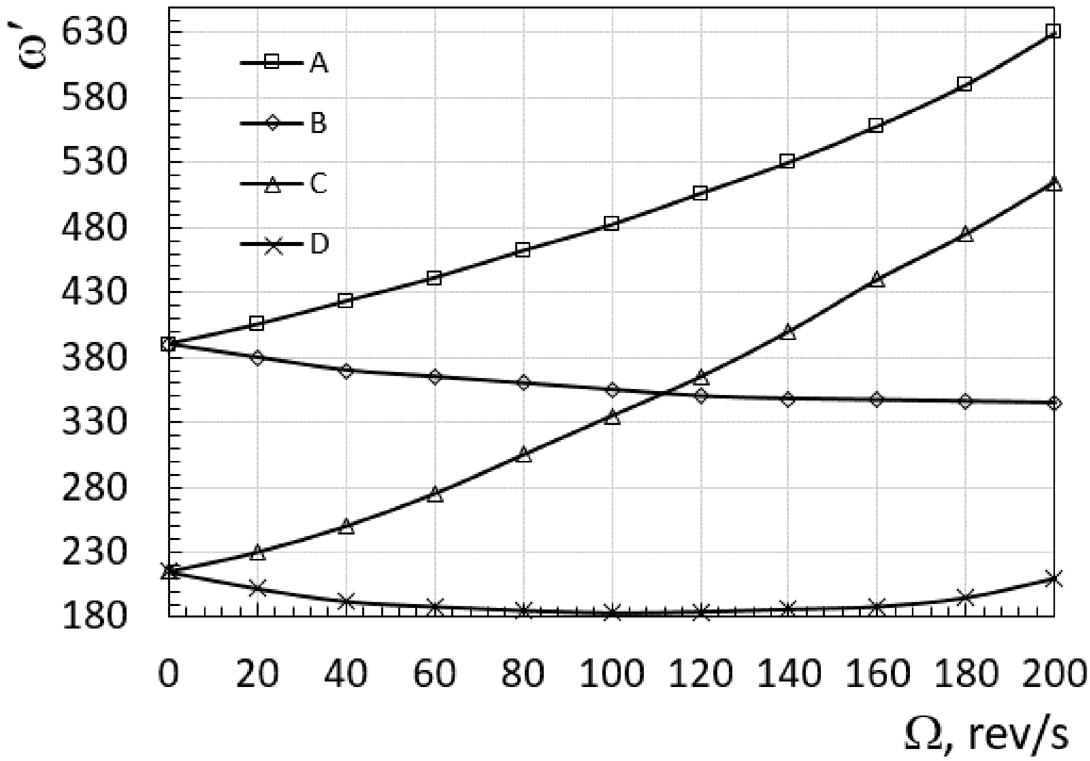


**Figure 2 - Variation of frequency parameter with rotating speed**

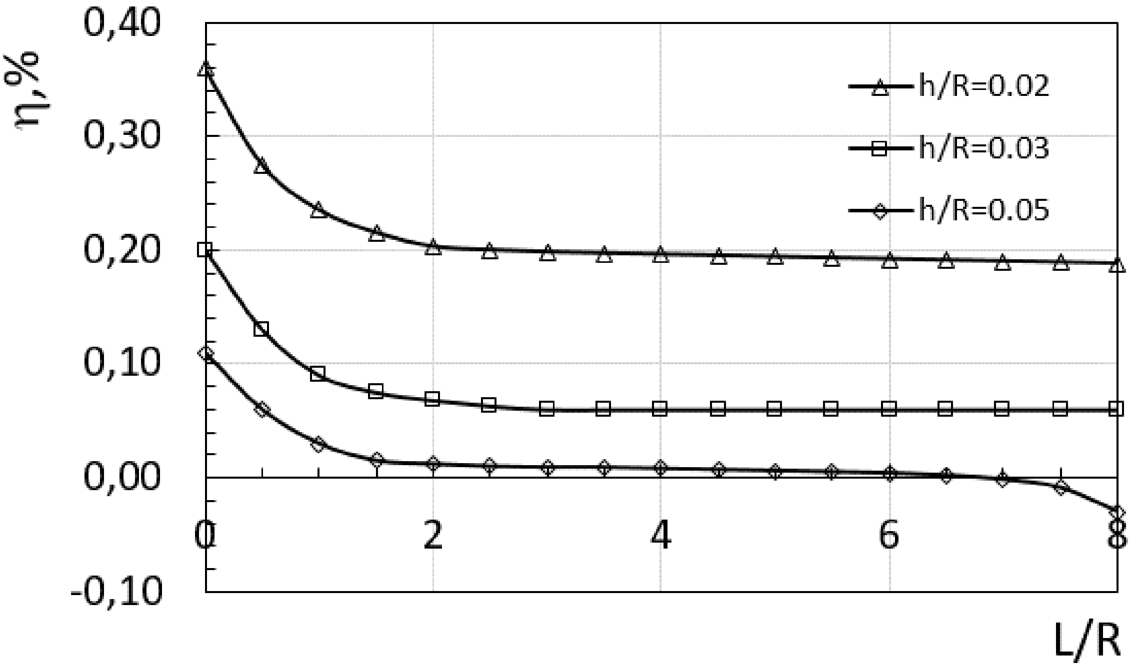


**Figure 3 - Variation of modal damping with length-to-radius ratio**

In addition, the damping value of the forward traveling wave of the rotating shell is always greater than the value of the backward traveling wave, indicating greater stability of the forward wave motion.

Figure 3 illustrates the curves of the modal damping with the length-to-radius ratio L/R for different thickness-to-radius ratios $h/R$, where the boundary condition A is taken as an example. The modal damping of the forward and backward traveling waves decreases with the increase of $h/R$ and sometimes decreases to negative values at certain parameters. In this case, the rotating shell will become unstable. It can be seen that increasing $h/R$ will not only obviously improve the damping characteristics, but also transform the unstable state of the system into a stable state. In addition, the modal damping curve drops sharply near the instability. This phenomenon may be due to the fact that the system is very sensitive to the change of parameters when it is close to the critical state.

**Summary and conclusions**.

The damping properties of rotating laminated composite cylindrical shells are investigated under four different boundary conditions with frequency-dependent damping of composite materials. The results of thin shell theory and the Haar wavelet discretization method allow obtaining the eigenvalue equation for the analysis of free vibrations.

The damping properties of rotating laminated composite cylindrical shells are investigated under four different boundary conditions with frequency-dependent damping of composite materials. The thin shell and Haar wavelet discretization techniques allow obtaining the eigenvalue equation for the analysis of free vibrations of the composite shell.

The numerical values of the attenuation coefficient of rotating composite cylindrical shells continuously increase with increasing frequency. The effect of rotation speed on modal attenuation is not the same for different circumferential wave numbers. The modal attenuation of the forward traveling wave of a rotating shell is greater than that of the backward wave, indicating that the forward wave motion is more stable. Both an increase and a decrease in modal attenuation with increasing layer angle were found. Attenuation corresponding to a smaller number of circumferential waves is more sensitive to changes in lamination patterns.

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