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Q-METHOD FOR NODAL VOLTAGE CONTROL IN ELECTRIC POWER SYSTEMS

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***Abstract*. Introductory speech on the research topic:** *The development of electric power systems is accompanied by the continuous growth of electricity generation and consumption, along with the increasing complexity of electric network configurations. It is becoming difficult to manage the modes of electrical systems. When managing system modes, it is necessary to calculate steady-state modes. As a result of the load flow solve, the permissible values of the mode parameters are determined. Maintaining permissible nodal voltage levels within the system becomes a priority. Significant voltage losses are caused by the passive parameters of transformers. Therefore, it is necessary to develop an efficient method for calculating steady-state modes to ensure effective voltage control.*

The purpose of scientific research: *Development and realization of the Q-method for controlling nodal voltages in the electric power system. A new computational method has been developed for solving the nonlinear algebraic equations of steady-state modes in systems with PU-type station nodes. The voltage levels at the nodes of the system are evaluated, taking into account the passive parameters of transformers. By applying the Q-method, permissible levels of nodal voltages are maintained through change in reactive power and phase angles of complex voltages, improving the system's economic efficiency.*

Description of scientific and practical significance of the work: *Scientific value of the work: The research introduces a computational Q-method for solving mixed equations of steady-state modes in electric systems with PQ- and PU-type station nodes. Practical significance: The method enables the control of node voltage levels in electric system while ensuring economic efficiency.*

Description of the research methodology: *Considering the continuous growth of electricity consumption, the complexity of network configurations, matrix theory, numerical methods for solving nonlinear algebraic equations of stabilized modes, voltage and reactive power modes, and voltage regulation measures, the Q-method for node voltage regulation in electric power systems has been developed.*

Main results, conclusions of the research work: *The study was conducted on a macromodel of the Armenian electric power system. The analysis indicates that the proposed Q-method is applicable to transmission electrical networks. The method evaluates changes in reactive power and phase angles of complex voltages at station nodes, improving system economic efficiency.*

The value of the conducted research (what contribution of this work to the relevant branch of knowledge): The presented method expands the scope of application for controlling and managing voltage levels at station nodes in operational mode calculations and analyses.

Practical significance of the results of work: The developed method allows solving problems of multi-mode control, analyzing and detecting violations of technical voltage limitations, and solving voltage regulation problems.

Keywords: *electric power system, nodal voltage, transformer, reactive power, method, matrix.*

Introduction.

The continuous changes in electrical energy consumption and the configurations of electrical networks complicate the management of electric power systems. The diverse mode problems necessitate the use of new methods for modeling steady-state modes.. Voltage mode in the electrical system significantly influence the transmission capacity of networks, the level of stability, losses, and the quality of electrical energy. Managing modes based on voltage and reactive power is a complex task. Voltage in the electrical system is constantly changing. The causes of voltage changes may include variations in the active and reactive loads of electrical consumers, voltage and reactive power of power plants, and changes in the passive parameters of elements in the network configuration (such as transmission lines and transformers). Therefore, the challenge arises to maintain the permissible voltage levels in the nodes of the electrical system. For the management of nodal voltages in the system, it becomes necessary to improve computational methods for solving non-linear algebraic equations of steady-state modes for PQ and PU type nodal points [1, 2] The integration of the method with the economic efficiency of the system and voltage regulation means is particularly emphasized.

Literature review. The system of non-linear algebraic equations for the steady-state mode of electric power systems is presented in Z-form [1–8]:

$$\dot{U} = \dot{U}_{0Б} + Z \cdot \dot{I} \tag{1}$$

(1) the system is expressed as follows:

$$\dot{U} = Z \cdot \dot{I}' \tag{2}$$

where:

Z- is the nodal impedance matrix of the circuit.

$$Z = Y^{-1} \tag{3}$$

Y-is the nodal admittance matrix of the circuit.

$\dot{I}'$- represents the complex currents of the independent nodes in the electrical system.

On the other hand, we obtain:

$$\dot{I}' = \frac{\hat{S}}{\hat{U}} - Y_0 \cdot U_0 \quad (4)$$

By substituting expression (4) into equation (2) and applying the following notation:

$$\dot{U}_{0Б} = Z \cdot Y_0 \cdot U_0 \quad (5)$$

We derive the following matrix equation:

$$\dot{U} = -\dot{U}_{0Б} + Z \cdot \frac{\hat{S}}{\hat{U}} \quad (6)$$

Writing equation (6) for the first node, we obtain:

$$\dot{U}_1 = Z_{11} \cdot \frac{\widehat{S_1}}{\widehat{U_1}} - (\dot{U}_{0Б} - Z_{12} \cdot \dot{I}_2{}' + \ldots + Z_{1M} \cdot \dot{I}_M{}') \quad (7)$$

Let us apply the following notation:

$$\dot{U}_1{}' = -(\dot{U}_{0Б} - Z_{12} \cdot \dot{I}_2{}' + \ldots + Z_{1M} \cdot \dot{I}_M{}') \quad (8)$$

Considering the notation in (8), equation (7) takes the following form:

$$\dot{U}_1 = Z_{11} \cdot \frac{\widehat{S_1}}{\widehat{U_1}} + \dot{U}_1{}' \quad (9)$$

For the k-th node, we derive:

$$\dot{U}_k = \dot{U}_{k0} - Z_{kk} \cdot \frac{\widehat{S_k}}{\widehat{U_k}} \quad (10)$$

where:

$$U_{k0} = U_{0Б} - \sum_{\substack{t=1 \\ t \neq k}}^{M} Z_{kt} \cdot \frac{\widehat{S_t}}{\widehat{U_t}} \quad (11)$$

For PU-type nodes, the equations to be solved are also written in the following forms:

$$Q_i = -\sum_{k=1}^{n} |V_i| \cdot |V_k| \cdot |V_{ik}| \cdot \sin(\theta_{ik} - \delta_i + \delta_k) \quad (12)$$

$$e_i^2 + f_i^2 = |V_i|^2 \quad (13)$$

$$e_i = \sqrt{|V_i|^2 - f_i^2} \quad (14)$$

where:

e_i- represents the real component of the complex voltage $\dot{V}_i$,

f_i- represents the imaginary component of the complex voltage $\dot{V}_i$.

The system (1) of equations is solved for PQ-type nodes, representing loads and power plants. Equations (10), (12), and (13) are solved for PU-type nodes of power plants. The solution methods for these equations do not always yield acceptable results. From this perspective, it is necessary to develop new computational methods for solving mixed equations of steady-state modes, which can be applied to the study of electrical systems. The new method is significant because it ensures the permissible voltage levels at the nodes of modernelectric power systems and enhances the economic efficiency of the system.

Aim of the Research. For the control of nodal voltages in an electric power system, it is proposed to use:

1. Calculation of a steady-state mode with PQ-type nodes considering the passive parameters of transformers.
2. Q-method for PU-type nodes (reactive power method)..
3. A criterion for assessing the economic efficiency of the electric power system.

Main Body. Let us assume that the electric power system (EPS) consists of M + 1 nodes (see Fig. 1.).

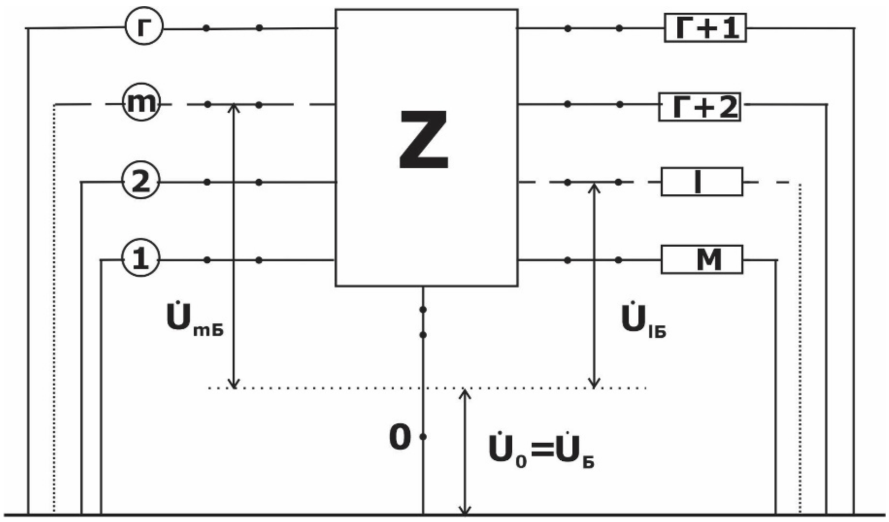

Figure 1 – Equivalent scheme of the EPS with the Z-form

It is supposed that the powers of the power plant nodes (0, 1, 2,..., Г) and load nodes (Г + 1, Г + 2,..., Г + Н = М) are given by PQ-type.

The electric power system consists of M+1 nodes. The node with index «0» is selected as the slack node. In this case, the equation of state of the electrical system in the Z-form takes the following form [9]:

$$\left.\begin{aligned} \dot{U}_1 &= \dot{U}_{0Б} + Z_{11}\cdot\dot{I}_1 + Z_{12}\cdot\dot{I}_2 + \ldots + Z_{1M}\cdot\dot{I}_M, \\ \dot{U}_2 &= \dot{U}_{0Б} + Z_{21}\cdot\dot{I}_1 + Z_{22}\cdot\dot{I}_2 + \ldots + Z_{2M}\cdot\dot{I}_M, \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots, \\ \dot{U}_M &= \dot{U}_{0Б} + Z_{M1}\cdot\dot{I}_1 + Z_{M2}\cdot\dot{I}_M + \ldots + Z_{MM}\cdot\dot{I}_M. \end{aligned}\right\} \quad (15)$$

where $\dot{U}_{0Б}$, $\dot{U}_1$, $\dot{U}_2$,..., $\dot{U}_M$-are complex voltages of nodes 0, 1,..., M of the electrical system,

$\dot{I}_1$, $\dot{I}_2$,...,$\dot{I}_M$-are complex currents of nodes 1,2,..., M of the electrical system,

Z_{12},..., Z_{1M}, Z_{21},..., Z_{2M},..., Z_{M1}- are mutual impedances of independent nodes of the electrical system,

Z_{11}, Z_{22},..., Z_{MM}- are self-impedances of independent nodes 1,2,..., M of the electrical system.

The system of equations (15) is also represented in the following form:

$$\dot{U}_i = U_{0Б} + \sum_{j=1}^{M} Z_{ij}\cdot\dot{I}_j\,, i = 1,2,\ldots,M \quad (16)$$

The nodal equation of the electrical system (15) in a compact form takes the form:

$$\dot{U} = \dot{U}_{0Б} + Z\cdot\dot{I}. \quad (17)$$

where $\dot{U}_{0Б}$ is a multidimensional slack voltage vector of the electrical system,

Z – nodal complex matrix of self and mutual impedances, due to the longitudinal and transverse passive parameters of power lines,

$\dot{U}$ – multidimensional vector of the complex voltage of independent nodes of the electrical system,

$\dot{I}$-multidimensional vector of the complex current of independent nodes of the electrical system.

Let's represent the equivalent scheme of the EPS (Fig. 1.) in the form (Fig. 2.).

Let us write the matrix equation of the steady-state for the equivalent scheme of the electric power system presented in (Fig. 2.), we will have [10]:

$$\dot{U}^{Tr} = \dot{U}_{0Б} + Z\cdot\dot{I}^{Tr} \quad (18)$$

where

$\dot{U}^{Tr}$ multidimensional complex voltages vector of electrical system independent nodes, taking into account the passive parameters of transformers,

$\dot{I}^{Tr}$- multidimensional complex currents vector of electrical system independent nodes, taking into account the passive parameters of transformers.

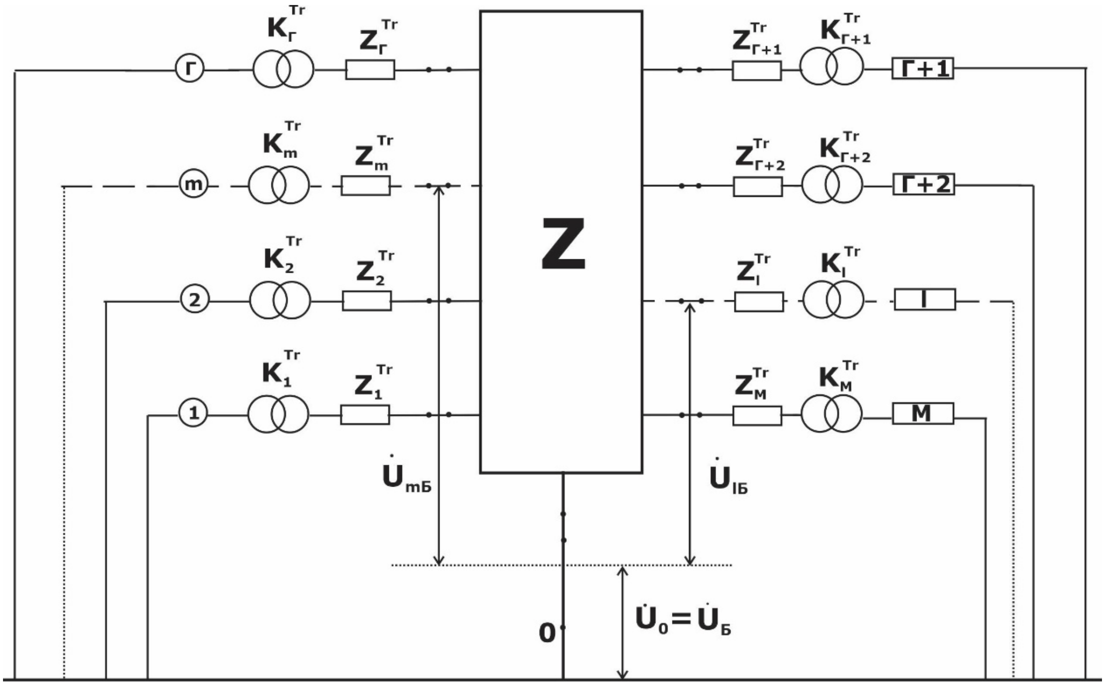

Figure 2 – Equivalent scheme EPS in Z-form, taking into account the parameters of transformers

For the application of the Q-method for controlling nodal voltages in an electric power system, PQ-type nodes are converted into PU-type nodes.

From matrix equations (16) and (17), we obtain:

$$\left|\dot{U}_i^{Tr}\right| \neq \left|\dot{U}_i\right| \tag{19}$$

where

$\dot{U}^{Tr}$ – is the multidimensional vector of the complex voltages of the independent nodes of the electric system, considering the passive parameters of transformers.

To satisfy condition (19), it is required to adjust the value of $\dot{U}_i^{Tr}$ voltage by $\Delta\dot{U}_i^{Tr}$, i. e.

$$\left|\dot{U}_i^{Tr} + \Delta\dot{U}_i^{Tr}\right| = \left|\dot{U}_i\right| \tag{20}$$

In this case, the increment of the complex voltage $\Delta\dot{U}_i^{Tr}$ is determined from expression (18) as follows. Voltage changes are made, causing a change in current, i. e.,

$$\dot{U}_i^{Tr} + \Delta\dot{U}_i^{Tr} = \dot{U}_{0Б} + \sum_{j=1}^{M} Z_{ij} \cdot (\dot{I}_J^{Tr} + \Delta\dot{I}_j^{Tr}) \quad (21)$$

from where

$$\Delta\dot{U}_i^{Tr} = \sum_{j=1}^{M} Z_{ij} \cdot \Delta\dot{I}_j^{Tr} \quad (22)$$

Substituting expression (22) into the left-hand side of formula (20) and performing the corresponding transformations, we get:

$$\left|\dot{U}_{i0}^{Tr} + Z_{ii} \cdot \Delta\dot{I}_i^{Tr}\right| = \left|\dot{U}_i\right| \quad (23)$$

where

$$\dot{U}_{i0}^{Tr} = \dot{U}_{0Б} + \sum_{\substack{i=1 \\ i \neq j}}^{M} Z_{ij} \cdot \dot{I}_j^{Tr} \quad (24)$$

Let us represent the left-hand side of expression (23) in the following form:

$$\left|(U_{ai0}^{Tr} + R_{ii} \cdot \Delta I_{ai}^{Tr} - X_{ii} \cdot \Delta I_{ri}^{Tr}) + j(U_{ri0}^{Tr} + R_{ii} \cdot \Delta I_{ri}^{Tr} + X_{ii} \cdot \Delta I_{ai}^{Tr})\right| = \left|\dot{U}_i\right| \quad (25)$$

where

$$U_{ai0}^{Tr} = Re(\dot{U}_{i0}^{Tr}) \quad (26)$$

$$U_{ri0}^{Tr} = Im(\dot{U}_{i0}^{Tr}) \quad (27)$$

$$\Delta I_{ai}^{Tr} = Re\left(\Delta\dot{I}_i^{Tr}\right) \quad (28)$$

$$\Delta I_{ri}^{Tr} = Im\left(\Delta\dot{I}_i^{Tr}\right) \quad (29)$$

Rewriting expression (25) in the following form, we obtain:

$$(U_{ai0}^{Tr} + R_{ii} \cdot \Delta I_{ai}^{Tr} - X_{ii} \cdot \Delta I_{ri}^{Tr})^2 + (U_{ri0}^{Tr} + R_{ii} \cdot \Delta I_{ri}^{Tr} + X_{ii} \cdot \Delta I_{ai}^{Tr})^2 = (\dot{U}_i)^2 \quad (30)$$

Performing transformations in expression (30) and neglecting the quadratic terms of current change ΔI^2, we get:

$$a_{ai}^U \cdot \Delta I_{ai}^{Tr} + a_{ri}^U \cdot \Delta I_{ri}^{Tr} = A_i^U \quad (31)$$

where a_{ai}^U, a_{ri}^U and A_i^U the coefficients are determined as follows:.

$$a_{ai}^U = 2 \cdot \left(R_{ii} \cdot U_{ai0}^{Tr} + X_{ii} \cdot U_{ri0}^{Tr}\right) \quad (32)$$

$$a_{ri}^U = 2 \cdot \left(R_{ii} \cdot U_{ri0}^{Tr} - X_{ii} \cdot U_{ai0}^{Tr}\right) \quad (33)$$

$$A_i^U = (\dot{U}_i)^2 - (U_{i0}^{Tr})^2 \quad (34)$$

The resulting equation (31) has two unknowns: the active component of current change ΔI_{ai}^{Tr} and the reactive component ΔI_{ri}^{Tr}. To determine these components, it is necessary to construct a second equation.

Since the node is assumed to be of PU type, for the given active power of the node, we can write the following formula:

$$P_i^{sub} = Re[(\dot{U}_{i0}^{Tr} + Z_{ii} \cdot \Delta\dot{I}_i^{Tr}) \cdot (\hat{I}_i^{Tr} + \Delta\hat{I}_i^{Tr})] \quad (35)$$

where $\hat{I}_i^{Tr} = I_{ai}^{Tr} - jI_{ri}^{Tr}$, $\Delta\hat{I}_i^{Tr} = \Delta I_{ai}^{Tr} - j\Delta I_{ri}^{Tr}$ - conjugate complex values of currents.

Performing the appropriate transformations in equation (35), neglecting the quadratic terms of current change ΔI^2, and taking the real part of the equation, we obtain:

$$a_{ai}^P \cdot \Delta I_{ai}^{Tr} + a_{ri}^P \cdot \Delta I_{ri}^{Tr} = A_i^P \quad (36)$$

where a_{ai}^P, a_{ri}^P and A_i^P the coefficients are determined by the following expressions:

$$a_{ai}^P = (U_{ai0}^{Tr} + R_{ii} \cdot I_{ai}^{Tr} + X_{ii} \cdot I_{ri}^{Tr}) \quad (37)$$

$$a_{ri}^P = (U_{ri0}^{Tr} + R_{ii} \cdot I_{ri}^{Tr} - X_{ii} \cdot I_{ai}^{Tr}) \quad (38)$$

$$A_i^P = P_i^{sub} - P_i \quad (39)$$

$$P_i = U_{ri0}^{Tr} \cdot I_{ai}^{Tr} + U_{ri0}^{Tr} \cdot I_{ri}^{Tr} \quad (40)$$

As a result, we obtain the following system of equations:

$$\left.\begin{aligned} a_{ai}^U \cdot \Delta I_{ai}^{Tr} + a_{ri}^U \cdot \Delta I_{ri}^{Tr} = A_i^U, \\ a_{ai}^P \cdot \Delta I_{ai}^{Tr} + a_{ri}^P \cdot \Delta I_{ri}^{Tr} = A_i^P. \end{aligned}\right\} \quad (41)$$

Representing the system of equations (41) in matrix form, we obtain:

$$\begin{bmatrix} a_{ai}^U & a_{ri}^U \\ a_{ai}^P & a_{ri}^P \end{bmatrix} \cdot \begin{bmatrix} \Delta I_{ai}^{Tr} \\ \Delta I_{ri}^{Tr} \end{bmatrix} = \begin{bmatrix} A_i^U \\ A_i^P \end{bmatrix} \quad (42)$$

From matrix equation (42), determine the active and reactive components of current changes.

$$\begin{bmatrix} \Delta I_{ai}^{Tr} \\ \Delta I_{ri}^{Tr} \end{bmatrix} = \begin{bmatrix} a_{ai}^U & a_{ri}^U \\ a_{ai}^P & a_{ri}^P \end{bmatrix}^{-1} \cdot \begin{bmatrix} A_i^U \\ A_i^P \end{bmatrix} \quad (43)$$

Substituting the current changes into formula (23), determine the magnitudes of the node's complex voltage $\dot{U}_i^{Tr}$ and argument ψ_{Ui}^{Tr}.

$$\psi_{Ui}^{Tr} = arctan\frac{U_{ri}^{Tr}}{U_{ai}^{Tr}} \quad (44)$$

In this case, the reactive power of the node is determined by the following formula:

$$Q_i^{sub} = Im[\dot{U}_i^{Tr} \cdot (\hat{I}_i^{Tr} + \Delta\hat{I}_i^{Tr})] \quad (45)$$

Performing certain transformations in formula (45) and taking the imaginary part, we obtain:

$$Q_i^{sub} =- U_{ai}^{Tr} \cdot \left(I_{ri}^{Tr} + \Delta I_{ri}^{Tr}\right) +. U_{ri}^{Tr} \cdot (I_{ai}^{Tr} + \Delta I_{ai}^{Tr}) \quad (46)$$

The iterative calculation process is organized by the following expression:

$$(\dot{U}_i^{Tr})^{\vartheta+1} = (\dot{U}_{i0}^{Tr})^{\vartheta} + Z_{ii} \cdot (\Delta \dot{I}_i^{Tr})^{\vartheta} \quad (47)$$

where

ϑ-is the iteration number.

The following condition is chosen as the criterion for completing the iterative calculation process:

$$\left.\begin{matrix} \left|\left(\dot{U}_i^{Tr}\right)^{\vartheta+1} - \dot{U}_\iota\right| \leq \varepsilon_{\Delta U_i,} \\ (tan\varphi_i^{Tr})^{\vartheta+1} \in \Delta tan\varphi_i^{d}. \end{matrix}\right\} \quad (48)$$

where

$\varepsilon_{\Delta U_i}$-is the convergence criterion,

$\Delta tan\varphi_i^{d}$-is the range of desired changes in the tangent of the node’s angle.

The relative magnitude of changes in approximate active power losses in the electrical system can be estimated by the following formula [11]:

$$\Delta\Pi_P = \Pi_P \cdot (1 - 0.02 \cdot K_{node}) \quad (49)$$

where

K_{node}- is the coefficient of increase in voltage level at stations nodes.

The changes in voltage and reactive power at nodes of the stations can be estimated using the Euclidean norm of the vectors [12], i. e.

$$\|\Delta U^{Tr}\|_2 = \sqrt{\sum_{i=1}^{M} \left|\Delta U_i^{Tr}\right|^2} \quad (50)$$

$$\left\|\Delta Q^{sub}\right\|_2 = \sqrt{\sum_{i=1}^{M} \left|\Delta Q_i^{Tr}\right|^2} \quad (51)$$

The study was carried out on the macromodel of the Armenian EPS. The simple iteration method is used to solve the steady-state equations. The results are presented in tables. The Hrazdan Thermal Power Plant (index «0») is represented as the slack node, the Yerevan Thermal Power Plant (index «1»), the Armenian Nuclear Power Plant (index «2») are the stations nodes.

Table 1 – Mode parameters of PQ-type nodes

node, i	P_i, MW	Q_i, MVAr	$\lvert\dot{U}_i\rvert$, kV	ψ_{Ui}, °	I_{ai}, kA	I_{ri}, kA	$tan\varphi_i$	$cos\varphi_i$
0			220	0				
1	240	148	211.974	-1.6777	0.6635	-0.3862	0.61	0.85
2	390	241	214.6895	-0.7986	1.0553	-0.6369	0.61	0.85
3	867	650	210.517	-2.262	-2.4384	1.6967	0.74	0.8
4	585	439	210.6054	-2.1348	-1.6418	1.1495	0.75	0.8

Table 2 – Mode parameters of PQ-type nodes in the presence of passive parameters of transformers

node, i	P_i^{sub}, MW	Q_i^{sub}, MVAr	$\lvert\dot{U}_i^{Tr}\rvert$, kV	ψ_{Ui}^{Tr}, °	I_{ai}^{Tr}, kA	I_{ri}^{Tr}, kA	$tan\varphi_i^{Tr}$	$cos\varphi_i^{Tr}$
0			220	0				
1	239.1444	119.9535	206.9752	-1.5842	0.676	-0.3164	0.5	0.89
2	388.2173	182.8176	208.9606	-0.6325	1.0779	-0.4937	0.47	0.9
3	873.2892	808.6308	206.3364	-2.2068	-2.528	2.1682	0.92	0.73
4	588.5188	545.8148	205.559	-2.0632	-1.7064	1.4735	0.92	0.73

Q-method

Table 3 – Mode parameters of PU-type nodes

node, i	P_i^{sub}, MW	Q_i^{sub}, MVAr	$\lvert\dot{U}_i^{Tr}\rvert$, kV	ψ_{Ui}^{Tr}, °	I_{ai}^{Tr}, kA	I_{ri}^{Tr}, kA	$tan\varphi_i^{Tr}$	$cos\varphi_i^{Tr}$
0			220	0				
1	239.1444	126.6430	211.9838	8.4941	0.6960	-0.2452	0.52	0.88
2	388.2173	197.4205	214.8231	14.9249	0.7586	-0.3476	0.5	0.89

Q-method

Table 4 – Coefficients of PU-type nodes

node, i	a_{ai}^U Ω· kV	a_{ri}^U Ω· kV	A_i^U kV^2	a_{ai}^P Ω· kA	a_{ri}^P Ω· kA	A_i^P MW
1	550.4698	-1679.6	-4.1662	209.1193	26.1641	100.9185
2	1059.1	-2466.7	-58.0189	206.9344	42.5407	249.9835

Conclusions.

1. The Q-method for controlling node voltages in the electrical power system ensures an acceptable voltage level by varying reactive powers and the angles of complex voltages: $\Delta Q_1^{sub} = 5.57\%$, $\Delta\psi_{U1}^{Tr} = 10.0783^{\circ}$, $\Delta Q_2^{sub} = 7.98\%$, $\Delta\psi_{U2}^{Tr} = 15.5574^{\circ}$.

2. The coefficient of voltage increase in the electrical power system nodes is:K_{node}=2.62%, with the variation in reactive power being:$\left\|\Delta Q^{sub}\right\|_2$=7.34%, and the

relative reduction in active power losses: $\Delta\Pi_P = 5.24\%$.

3. The Q method allows to control the voltage level in the nodes of the electric power system by regulating the change in reactive power and increasing the economic efficiency of the system.

Prospects for further research.

1. Study of the Q-method in system-forming electric networks.

2. Research on voltage regulation in transmission networks with economic efficiency.

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