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LAMB WAVE-BASED STRUCTURAL MONITORING IN COMPOSITE LAMINATES

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Abstract. *This study presents a numerical analysis of a quasi-isotropic layup, which serves as a working model for assessing the level of anisotropy in laminated composites. To achieve this, Lamb wave propagation was simulated in four distinct directions: 0°, 30°, 60°, and 90° relative to the laminate's x-axis. The primary assumption for this structural monitoring model is that the composite is quasi-isotropic throughout its entire volume. Our calculations of the dispersion curves revealed the presence of a cut-off frequency, which defines the boundaries of a non-dispersive region. Within this region, only three fundamental Lamb wave modes* S_0*,* A_0*, and* SH_0 *can exist. The velocity of the Lamb wave packets remains nearly constant in this range, with the exception of the A0 wave near the origin. It should be noted that the level of laminate anisotropy is determined by its quasi-isotropic stacking sequence. It was found that the characteristic value of the* A_0 *group velocity is superimposed on fluctuations. To extract the group velocity values, the amplitude of the Lamb wave packets corresponding to the midpoint of the laminate volume was processed using the Hilbert transform. The basis for determining this velocity is the wavenumber values calculated using a reference value of the angular frequency. The results demonstrate that the proposed approach is an effective tool for analyzing and evaluating the anisotropy of composite materials.*

Key words: *numerical analysis, laminated composites, Lamb waves, anisotropy, dispersion curves, Hilbert transform.*

Introduction.

Structural health monitoring (SHM) is a crucial area of research in materials science and engineering. It involves the use of sensing technologies to assess the condition of a structure, detecting damage, and predicting its remaining service life. A notable challenge in this field is the complexity of composite laminates, which are widely used due to their high strength-to-weight ratio and customizable properties. These materials, however, are susceptible to various forms of damage, including delamination, matrix cracking, and fiber breakage, which are often difficult to detect using traditional non-destructive testing (NDT) methods. This difficulty arises from the anisotropic nature of composites and the subtle, internal character of the damage mechanisms.

Among the various NDT techniques, guided wave propagation has emerged as a promising approach for the in-situ monitoring of large-area composite structures.

Unlike conventional ultrasonic testing, which uses bulk waves for localized inspection, guided waves can travel long distances within a structure, allowing for the interrogation of a large volume of material from a single point. This characteristic makes them particularly suitable for continuous monitoring systems. Within the family of guided waves, Lamb waves are of particular interest for SHM applications in plate-like structures, such as composite laminates. Lamb waves are elastic waves that propagate in thin plates and exhibit a complex multimodal and dispersive nature. Their propagation characteristics, including velocity and attenuation, are highly sensitive to the presence of damage.

The propagation of Lamb waves in composite laminates is a complex phenomenon governed by the material's elastic properties and lay-up configuration. The anisotropic nature of the material causes the wave velocity to vary with direction, a property that can be exploited for damage localization. Moreover, the dispersive nature of Lamb waves, where different frequencies travel at different velocities, presents both a challenge and an opportunity. While it complicates signal analysis, it also provides multiple data points for damage detection, as different modes and frequencies interact with damage in distinct ways. The presence of damage, such as delamination, introduces discontinuities in the wave path, leading to scattering, reflection, and mode conversion of the guided waves. By analyzing these changes in the wave signal, it is possible to infer the location, size, and type of damage.

The implementation of a Lamb wave-based SHM system involves several key components. The system typically consists of a network of piezoelectric transducers that act as both actuators to generate the waves and sensors to receive the propagated signals. The signals are then processed using sophisticated algorithms to extract damage-sensitive features. These algorithms often employ signal processing techniques like wavelet transforms, and data analysis methods to correlate changes in the received signal with the presence and characteristics of damage. The development of advanced signal processing and data interpretation techniques is crucial for moving from simple damage detection to more quantitative assessments, such as sizing and classification. This requires overcoming challenges related to environmental and operational variations, which can also influence wave propagation and introduce false alarms.

Research in this area focuses on several key aspects, including the fundamental understanding of Lamb wave-damage interaction mechanisms in various composite lay-ups. This involves developing accurate numerical models to simulate wave propagation and validate experimental findings. Another important area is the development of advanced signal processing algorithms that can robustly detect and localize damage in the presence of noise and environmental variability. The design of optimized sensor networks to maximize coverage and sensitivity is also a critical consideration. Furthermore, the integration of these systems into a complete, real-time monitoring framework is a major goal, aiming to provide continuous and autonomous structural assessment. The ultimate objective is to move from reactive maintenance to a predictive, condition-based approach, enhancing the safety and reliability of critical structures. The continuous development of these techniques holds the potential to significantly improve the safety and operational efficiency of composite structures.

The goal of this study is to conduct a detailed numerical analysis of experimental data obtained during the process of Lamb wave scattering. This analysis will be used to evaluate the integrity of the laminated composite joint. We propose to use matrix analysis of the results of numerical calculations, which allows for a comprehensive assessment of the structure's condition, revealing hidden defects and predicting its durability.

Ultrasonic guided waves have proven to be an excellent experimental tool for non-destructive testing and investigation of adhesive bonds between composite laminates [1]. They provide information about the material's condition without causing damage, which is critically important for high-tech structures. The study of the transient dynamics and wave propagation characteristics in adhesive composite joints is most effective using the wavelet spectral finite element model. This method accurately models wave processes, taking into account the complex geometry and anisotropy of composite materials, and also detects subtle changes in the signal caused by the presence of defects.

Defect detection in common types of composite joints, such as T-joints and L-joints, can also be effectively performed using guided Lamb waves. These joints are frequently used in the aerospace, automotive, and shipbuilding industries, and their reliability directly affects the safety and performance of the final products. Traditional inspection methods are not always capable of revealing internal defects, such as delaminations or loss of adhesion, which can lead to catastrophic failure.

However, despite significant progress, Lamb wave-based damage monitoring technologies for local mechanical joints in composites are still not well developed. There is a pressing need for a deeper investigation into this matter to ensure reliable and accurate inspection. It is of particular interest to investigate local composite joints based on the phenomenon of Lamb wave propagation [2]. This will enable the development of new methods and algorithms for detecting defects specific to these types of structures.

Specifically, within the framework of this study, we consider it appropriate to examine two different types of local joints in a composite structure that differ in their length-to-width ratio. This approach will allow us to assess how geometric parameters influence the propagation of Lamb waves and, consequently, the effectiveness of defect detection. Such an analysis will help create more universal and reliable monitoring systems capable of working with various joint configurations. The results of our research will contribute to the development of the field of structural health monitoring and non-destructive testing, ultimately increasing the safety and longevity of composite structures.

Stress-strain and load-displacement curves technique.

Mechanical effects on the surface of the composite sample naturally lead to a change in the deformation field and, accordingly, to a change in the characteristics of the Lamb waves scattered by defects. The propagation behavior of Lamb waves in joints, especially in the region of high mechanical local joint concentrations, was analyzed using three-dimensional finite element modeling. The numerical values of the parameters in the modeling were specified using the available results of the baseline experiments.

The results of numerical calculations were accumulated in the average stress-strain and load-displacement curves of the composite laminates for different types of mechanical loading. The stress-strain and compression curves (σ_n - ε_n) along 90^0, 0^0 and $\pm45^0$ fibers of the laminar composites were calculated according to the three-point bending test. Besides, the normalized (index "*n*" in curves) load-displacement curves (P_n - δ_n) were obtained in multimode tests.

Modes A, B and C are characterized by the following regimes: the origin of localized deformations in the form of cracks (mode A); the development of multiple cracks to their maximum geometric dimensions (mode B); the initial stage of the composite sample delamination caused by a set of developed cracks (mode C).

One of the quite effective methods for numerical modeling of the propagation of guided Lamb waves in composite laminates of arbitrary layup and cross-sectional geometry is the semi-analytical finite element method. It should be noted that the traditional finite element method in application to laminated composites is computationally expensive and may lead to numerical failure, especially in the case of short wavelengths.

The semi-analytical finite element method uses a finite element two-dimensional discretization of the cross-sectional area. The basic assumption in this model is that the displacements along the direction of propagation of the wave packet are assumed to have the form of a harmonic wave and are plane-polarized. Moreover, the typical planar geometry of laminates allows for further simplification in terms of one-dimensional modeling.

The calculation procedure considers the cross-sectional domain Ω of the laminated composite specimen, which is represented by a finite element system with domain Ω_e

$$u(x,y,z,t)=\begin{bmatrix} u_x(x,y,z,t) \\ u_y(x,y,z,t) \\ u_z(x,y,z,t) \end{bmatrix}=\begin{bmatrix} u_x(y,z) \\ u_y(y,z) \\ u_z(y,z) \end{bmatrix}\exp[i(\xi x-\omega t)], \tag{1}$$

where ω is the angular temporal frequency.

The field of local displacements anisotropically located along the volume of the laminated composite is assumed to be harmonic along the x - propagation direction. The characteristics of the displacement locations are described by spatial functions that are used to describe its amplitude in the y - z cross-sectional plane (1).

When using one-dimensional elements, a separate procedure was adopted for discretizing Ω. The discretized version of the displacement expressions over the element domain can be written in terms of shape functions, N_k (y, z), and unknown nodal displacements, (U_{xk}, U_{yk}, U_{zk}) in the x, y, and z directions:

$$u^e(x,y,z,t)=\begin{bmatrix}\sum_{j=1}^{n} N_j(y,z)U_{xj} \\ \sum_{j=1}^{n} N_j(y,z)U_{yj} \\ \sum_{j=1}^{n} N_j(y,z)U_{zj}\end{bmatrix}\exp[i\xi x-\omega t]=N(y,z)q^e\exp[i\xi x-\omega t]. \quad (2)$$

Nodal displacements can be considered as arguments of some deformation vector

$$\varepsilon^e=\left[L_x\frac{\partial}{\partial x}+L_y\frac{\partial}{\partial y}+L_z\frac{\partial}{\partial z}\right]N(y,z)q^e\exp[i(k x-\omega t)]=$$

$$=(B_1+ikB_2)q^e\exp[i(k x-\omega t)], \quad (3)$$

where

$$B_1=L_yN_y+L_zN_z, \quad (4)$$

$$B_2=L_xN. \quad (5)$$

At the next stage, the technique requires writing down the discrete form of the formulation of Hamilton's equation (denoting by the symbol n_{el} the total number of elements of the cross section)

$$\int_{t_1}^{t_2}\left\{\bigcup_{e=1}^{n_{el}}\left[\int_{V_e}\delta(\varepsilon^{eT})C_e\varepsilon^e dV_e+\int_{V_e}\delta(u^{eT})\rho_e\ddot{u}^e dV_e\right]\right\}dt=0, \quad (6)$$

where

C_e is the complex stiffness matrix;

ρ_e is the density.

This numerical methodology assumes that the element stiffness matrix can be calculated by integrating only over the cross-sectional area Ω_e, since integration over x reduces to a unit factor due to the complex conjugate terms exp [$\pm\, i\,(k{\cdot}x - \omega t)$]. For viscoelastic materials, the strain energy consists of a real and an imaginary component. The real component of the strain energy describes the time-averaged elastic energy in the cross-section. The imaginary component of the strain energy is related to the time-averaged power dissipated by the cross-section.

Applying the standard finite element procedure to equation (6), we get

$$\int_{t_1}^{t_2} \left\{ \delta U^T \left[K_1 + ikK_2 + k^2 K_3 - \omega^2 M \right] U \right\} dt = 0\,, \tag{7}$$

where U is the global vector of nodal displacement and

$$K_1 = \bigcup_{e=1}^{n_{el}} k_1^e,\quad K_2 = \bigcup_{e=1}^{n_{el}} k_2^e,\quad K_3 = \bigcup_{e=1}^{n_{el}} k_3^e,\quad M = \bigcup_{e=1}^{n_{el}} m^e\,. \tag{8}$$

The homogeneous general wave equation has the form

$$\left[K_1 + ikK_2 + k^2 K_3 - \omega^2 M \right]_M U = 0\,, \tag{9}$$

where M is the number of total degrees of freedom of the system.

The stiffness matrices K_1 and K_3 in the equation are symmetric. The matrix K_2 is skew-symmetric and is related to the case where undamped motion is considered. For damped motion, all matrices K_i are usually complex. The matrix M is real symmetric and positive definite regardless of the type of motion (damped or undamped).

The matrix K_1 is related to the strain transformation matrix B_1, which is related to generalized plane strains. It should be noted that the matrix B_1 describes the generalized plane strain behavior or transverse buckling. The matrix K_3 describes the out-of-plane strain behavior since it depends on the matrix B_2. The matrix K_2 contains the matrices B_1 and B_2 and thus relates transverse buckling to out-of-plane strains.

Results in terms of multiple modes and dispersion properties can be obtained in a numerically stable manner using the eigenvalue and eigenvector problem. Two quadratic finite elements were used on a model example of a laminar composite containing 18 plates. Each element has three degrees of freedom per node, associated with mechanical displacements U_x, U_y, U_z.

Figures 1 shows the dispersion curves for the phase velocities $Cp = Cp$ (f). Considering the quasi-isotropic stacking and to quantify its anisotropy level, four different Lamb wave propagation directions (0°, 30°, 60°, and 90° relative to the x-coordinate of the laminate) were simulated. The basic assumption for the structure monitoring model is quasi-isotropy throughout the volume of the laminated composite.

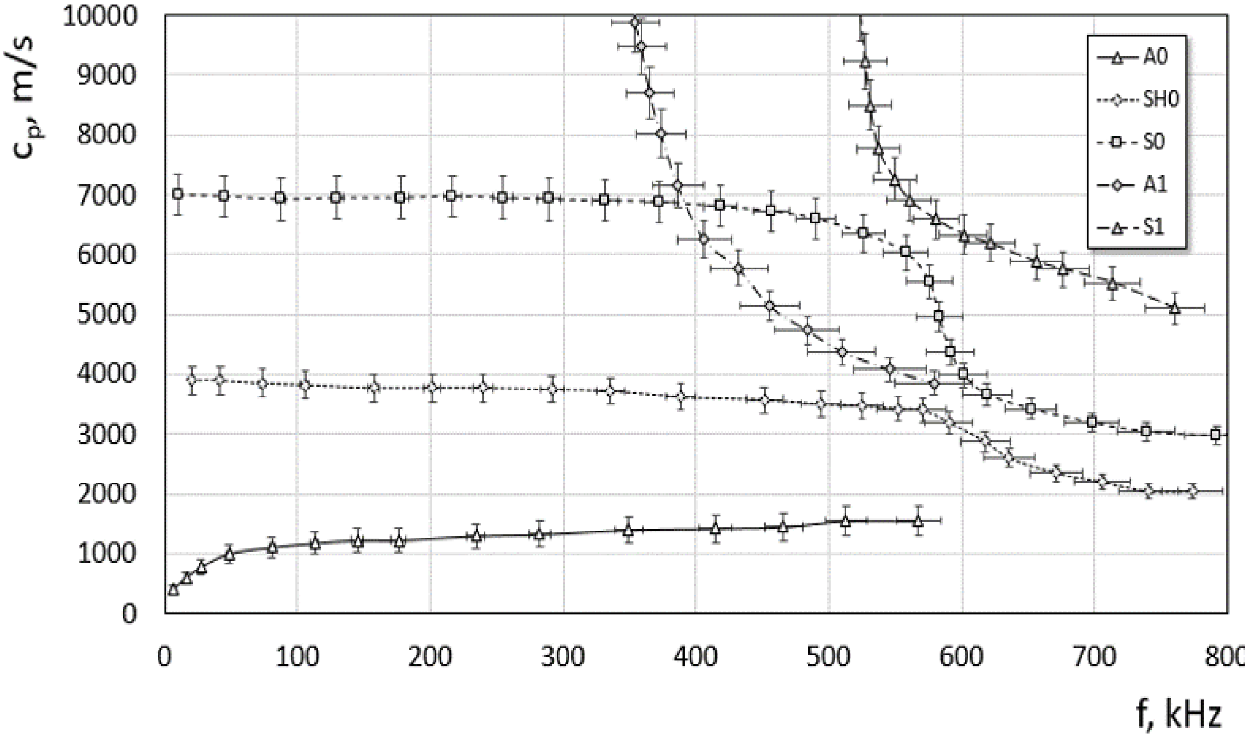

Figure 1 – Dispersion curve $c_p = c_p$ (f) for θ = 30⁰

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Summary and conclusions.

The calculations of the dispersion curve revealed the presence of a so-called cutoff frequency, which delineates the non-dispersive region. In this specific region, only three fundamental Lamb wave modes S_0, A_0, and SH_0 are able to exist and propagate effectively. A key characteristic of this region is that the velocity of the Lamb wave packets remains nearly constant, with the notable exception of the A_0 wave close to the origin. This non-dispersive behavior is highly advantageous for structural health monitoring applications because it simplifies the interpretation of the received signals. It's important to note that the level of anisotropy in the laminate is directly determined by the specific quasi-isotropic stacking sequence used. This sequence is a fundamental parameter that influences the mechanical and dynamic properties of the composite.

It was found that the characteristic value of the A_0 group velocity is superimposed on existing fluctuations. To accurately extract this velocity, the amplitude values of the Lamb wave packets, corresponding to the coordinate located at the midpoint of the laminate volume, were processed using a Hilbert transform.

References:

1. Putkis, O., Dalton, R. P., & Croxford, A. J. (2016). The anisotropic propagation of ultrasonic guided waves in composite materials and implications for practical applications. Ultrasonics, issue 65, pp. 390-399.
DOI: 10.1016/j.ultras.2014.11.013

2. Yang, C., Ye, L., Su, Z., & Bannister, M. (2006). Some aspects of numerical simulation for Lamb wave propagation in composite laminates. Composite structures, issue 75(1-4), pp. 267-275.
DOI: 10.1016/j.compstruct.2006.04.034

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